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Non-linear height-diameter models for oriental beech (*Fagus orientalis* Lipsky) in the Hyrcanian forests, Iran

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The relationship between tree height and diameter is an important element in growth and yield models, in carbon budget and timber volume models, and in the description of stand dynamics. Six non-linear growth functions (*i.e.* Chapman-Richards, Schnute, Lundqvist/Korf, Weibull, Modified Logistic and Exponential) were fitted to tree height-diameter data of oriental beech in the Hyrcanian mixed hardwood forests of Iran. The predictive performance of these models was in the first place assessed by means of different model evaluation criteria such as adjusted R squared (adj R^2), root mean square error (*RMSE*), Akaike information criterion (*AIC*), mean difference (*MD*), mean absolute difference (*MAD*) and mean square (*MS*) error criteria. Although each of the six models accounted for approximately 75% of total variation in height, a large difference in asymptotic estimates was observed. Apart from this, the predictive performance of the models was also evaluated by means of cross-validation and by splitting the data into 5-cm diameter classes. Plotting the *MD* in relation to these diameter at breast height (*DBH*) classes showed for all growth functions, except for the Modified Logistic function, similar mean prediction errors for small- and medium-sized trees. Large-sized trees, however, showed a higher mean prediction error. The Modified Logistic function showed the worst performance due to a large model bias. The Exponential and Lundqvist/Korf models were discarded due to their showing biologically illogical behavior and unreasonable estimates for the asymptotic coefficient, respectively. Considering all the above-mentioned criteria, the Chapman-Richards, Weibull, and Schnute functions provided the most satisfactory height predictions. However, we would recommend the Chapman-Richards function for further analysis because of its higher predictive performance.
Keywords. Forest trees, *Fagus orientalis*, simulation models, growth, Iran.

Modèles non linéaires de diamètre de hauteur pour le hêtre oriental (*Fagus orientalis* Lipsky) dans les forêts Hyrcaniennes en Iran. La relation entre la hauteur des arbres et le diamètre est un élément important pour les modèles de croissance, de rendement, du budget de carbone et de volume du bois, et pour la description de la dynamique des peuplements. Six fonctions de croissance non linéaires (Chapman-Richards, Schnute, Lundqvist/Korf, Weibull, fonctions logistiques et exponentielles modifiées) ont été ajustées aux données de diamètre de hauteur des arbres de hêtre oriental dans les forêts mélangées hyrcaniennes d'Iran. La performance prévue des modèles a été évaluée à l'aide du R^2 ajusté (adj R^2), de l'erreur quadratique moyenne (*RMSE*), du critère d'information d'Akaike (*AIC*), de la différence moyenne (*MD*), de la différence absolue moyenne (*MAD*) et de l'erreur quadratique moyenne (*MS*). Les résultats ont montré que chacun de ces six modèles représente environ 75 % de la variation totale de hauteur, mais produit différentes estimations asymptotiques. La performance prévue a également été évaluée à l'aide des validations croisées et par séparation des données en classes de 5 cm de diamètre à hauteur de poitrine (*DBH*) afin de calculer le *MD* pour chaque classe. Les visualisations de *MD* pour toutes les classes *DBH* ont montré que les six fonctions de croissance, sauf la logistique modifiée, produisent des erreurs de prédiction moyennes similaires pour les arbres de tailles petites et moyennes. Cependant, pour les arbres de grande taille, l'erreur de prédiction moyenne est plus élevée. La fonction de logistique modifiée est la moins performante, en raison d'un large biais. Les modèles exponentiels et de Lundqvist/Korf ont été rejetés en raison, respectivement, de leur comportement biologique illogique et des estimations déraisonnables pour les coefficients asymptotiques. En envisageant tous les critères mentionnés ci-dessus, les fonctions Chapman-Richards, Weibull et Schnute fournissent les prédictions de hauteur les plus satisfaisantes, mais la fonction de Chapman-Richards pourrait être recommandée pour une analyse plus approfondie en raison de sa meilleure performance.
Mots-clés. Arbre forestier, *Fagus orientalis*, modèle de simulation, croissance, Iran.

1. INTRODUCTION

Covering an area of approximately 1.85 million ha, the Hyrcanian forests account for approximately 15% of Iranian forests and 1.1% of the country's total area. These forests range from sea level up to an elevation of 2,800 m and comprise various forest types including no less than 80 woody species (trees and shrubs). Oriental beech (*Fagus orientalis* Lipsky), oak (*Quercus castaneifolia* Coss. ex J.Gay), maple (*Acer velutinum* Boiss.), hornbeam (*Carpinus betulus* L.) and alder (*Alnus subcordata* C.A.Mey.) are among the main tree species in these forests. The Hyrcanian forests have been forestland since the third geological era and are considered to be one of the oldest forests in the world (Sagheb-Talebi et al., 2004).

The relationship between tree height and diameter at breast height (*DBH*) is one of the most important components of forest structure. Estimations of timber volume, site index, succession, carbon sequestration (Spurr, 1952; Botkin et al., 1972; Kurz et al., 1992; Vanclay, 1994; Peng et al., 2001), as well as stand description and damage appraisals (Parresol, 1992; Zhang, 1997) are highly related to the tree height-*DBH* relationship. Being one of the most commonly measured parameters in forest inventories, *DBH* is easily measured with little investment of time and cost and with a high level of accuracy. By contrast, the necessary investment of time, the chance of observer error and the occurrence of visual obstacles are among the main difficulties in measuring tree height (Colbert et al., 2002; Krisnawati et al., 2010). Therefore, in most forest inventories, the heights of only a few trees in a sample plot are measured, whereas the *DBHs* of all trees are measured. Consequently, height-*DBH* equations are very useful for the prediction of missing and unmeasured tree heights from field measurements and for numerous forest growth simulators (Huang et al., 2000; Peng et al., 2001; Lumbres et al., 2011). Furthermore, vertical forest structure can be well analyzed using height-*DBH* equations (*e.g.*, Wykoff et al., 1982; Van Deusen et al., 1985; Larsen et al., 1987; Ritchie et al., 1986; Larsen, 1994, Colbert et al., 2002).

A wide variety of models have been proposed for height-diameter relationships for different species and different forest regions. As Krisnawati et al. (2010) state, the approaches used for modeling height-diameter vary from linear to non-linear models. Curtis (1967) compared thirteen height-diameter models using linear regression techniques. On the other hand, Huang et al. (1992) selected the most appropriate height-diameter functions for major tree species out of 20 weighted non-linear techniques. These authors observed that the Weibull, the Modified Logistic, the Chapman-Richards and the Schnute functions generally provided the most satisfactory results. Zhang (1997) cross-validated six non-linear growth functions fitted to the tree height-diameter data of ten conifer species collected in the inland northwest of the United States. He concluded that the Schnute, Weibull, and Chapman-Richards functions presented the best predictive performance. Fang et al. (1998) investigated 33 height-diameter equations for tropical forests on Hainan Island in southern China. Peng et al. (2001) fitted six commonly used non-linear growth models to the tree height-diameter data of nine major tree species in Ontario's boreal forests. The results showed that the Chapman-Richards, Weibull, and Schnute functions provided the most satisfactory height predictions based on predictive performance criteria. Sánchez et al. (2003) used 26 linear and non-linear height-diameter functions for *Pinus radiata* D.Don throughout Galicia in the northwest of Spain and found that the Tomé model (Tomé, 1989) resulted in the best height estimates. Lumbres et al. (2011) developed and validated height diameter models for the three *Pinus* and one *Larix* species in South Korea using the six widely used non-linear growth functions. They showed that the Modified Logistic and Lundqvist/Korf models performed best compared to the other models based on a rank analysis. Pormajidian (1992) and Siahipour et al. (2002) have recommended non-linear models of height-diameter for *Picea abies* (L.) H.Karst. afforestation in Kelardasht region and Guilan province (in the north of Iran). In order to determine the most appropriate relationship between the diameter and height of *Picea abies* in Kelardasht afforestation (in the north of Iran), Fallah (2009) fitted 17 non-linear models to these data and selected the most satisfactory model based on *MS* error and R^2.

One of the most abundant and economic valuable hardwood genera in Hyrcanian forests is the *Fagus* genus. Beech forests account for approximately 17.6% of the total forest area, 30% of the standing volume and 23.6% of the stem number in the Hyrcanian forests in Iran. The average beech volume per ha varies between 480 and 740 cubic meters in pure stands and 600 and 700 cubic meters in mixed stands (Sagheb-Talebi et al., 2004). Because of the importance of oriental beech (*Fagus orientalis*) as one of the main timber species of the Hyrcanian forests, the aim of this study was to fit six commonly used non-linear growth models to height-diameter data of beech collected in the Tarbiat Modares University forest and to select the best model based on different evaluation criteria.

2. MATERIALS AND METHODS

2.1. Study site

This research was conducted in the Tarbiat Modares University (TMU) forest, a temperate forest forming part of the Hyrcanian forests, located in the Mazandaran province in the north of Iran, between 36° 31’ 56” N and 36° 32’ 11” N latitudes and 51° 47’ 49” E and 51 ° 47’ 56” E longitudes (**Figure 1**). The study area is approximately 300 ha and its elevation varies from 1,000 m to 1,500 m above sea level. The minimum temperature in December is 6.6 °C, and the highest temperature, 25 °C, occurs in June. The mean annual precipitation of the study area is approximately 1,500 mm, measured at the Nowshahr city meteorological station, which is located 40 km away from the study area. The study area consists of mixed and uneven-aged forests, dominated by *Fagus orientalis* associated with *Carpinus betulus*, *Acer velutinum*, *Parrotia persica* C.A.Mey., *Sorbus torminalis* (L.) Crantz, *Quercus castaneifolia*, *Alnus subcordata*, *Acer laetum* C.A.Mey., *Prunus avium* (L.) L., *Ulmus glabra* Huds. and *Tilia begoniifolia* Steven species. The forest is managed following close-to-nature principles with single selection harvesting techniques. The bedrock is mainly limestone-dolomite with a silty-clay-loam soil texture (Kooch et al., 2010).

2.2. Data, models and methods

The data used for modeling the height-*DBH* relationships were collected in 2012 using a random-systematic network of temporary sample plots. The interval between the grid lines was set at 200 and 100 m longitude and latitude, respectively. Two grid lines out of six were selected after field inspection. A total of 43 gridline intersections coinciding with plot centers were located in the field by GPS (Garmin 76 CSX) navigation. Plots with no evidence of disturbance including forest harvesting were discarded. These plots were circular-shaped and 0.1 ha in size. The plots extended the altitudinal belt ranging from 1,000 to 1,500 m a.s.l.

All trees with a *DBH* > 7 cm within the plots were measured for diameter at breast height (*DBH*) using a caliper to the nearest millimeter and the total height of

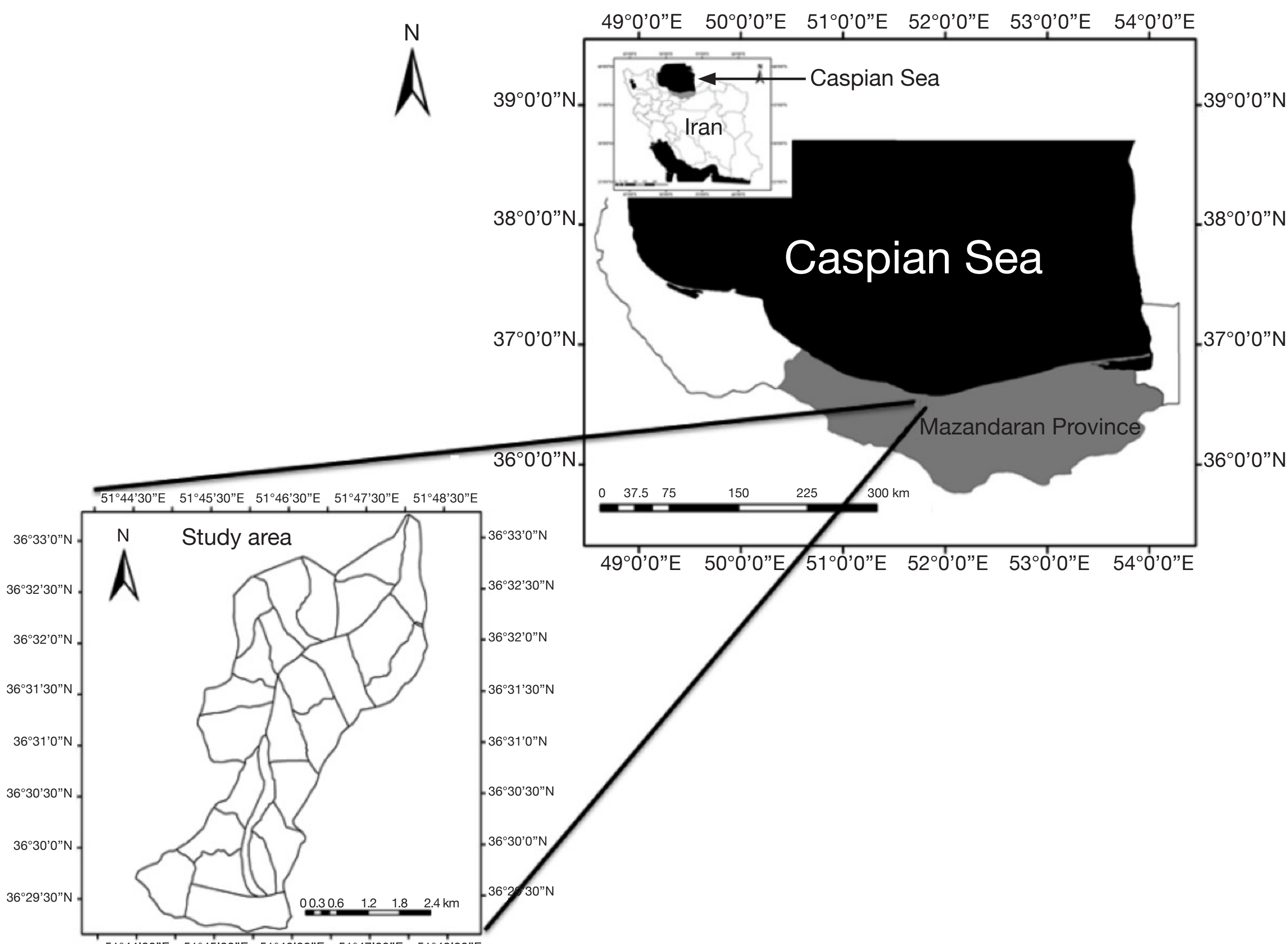

Figure 1. Study area — *Région d’étude*.

all beech trees was measured using a Vertex IV height meter. Multi-stemmed and damaged-top trees were not included in the analysis. A total of 605 individual height-diameter measurements for beech trees were recorded in the study area. The available tree height-diameter data were separated into two sets based on the grids: grid line 1 (n = 315) and grid line 2 (n = 290) were used for model calibration and model validation, respectively. Both datasets covered approximately the same ranges of *DBH* and height (**Figure 2**). Summary statistics for these two datasets are provided in **table 1**.

A wide variety of non-linear models are often recommended for modeling the relationships between tree height and *DBH* (for example, Huang et al., 1992; Moore et al., 1996; Zhang, 1997; Fang et al., 1998; Fekedulegn et al., 1999; Peng, 1999; Peng et al., 2001). Based on these studies, six non-linear growth functions (**Table 2**) were selected as candidate height-diameter models. These six non-linear growth functions have been widely used in the literature due to their appropriate mathematical properties and promising predictive performances for tree height-diameter relationships (Brewer et al., 1985; Arabatzis et al., 1992; Huang et al., 1992; Zeide, 1993; Zhang et al., 1996; Zhang, 1997; Fang et al., 1998; Fekedulegn et al., 1999; Huang, 1999). All of these six non-linear models present an asymptotic behavior (parameter *a* in the models) in terms of the maximum tree height.

The NLIN procedure in the Statistical Analysis System (SAS Institute Inc., 2002) was used to fit the six candidate models (**Table 2**) from the calibration dataset. This procedure applies an iterative process depending on the starting values for the parameters of the model being provided. The Gauss-Newton, the Marquardt, and the steepest descent methods are the three main iterative methods in the NLIN procedure. When parameter estimates of the model are highly correlated, the Marquardt method is considered to be the most useful (Fang et al., 1998). This method was therefore used in this study. Multiple initial values for the model parameters were used to guarantee that the least-squares solution was global rather than local. The homogeneity of variance assumption was investigated by plotting studentized residuals against the predicted height. This plot showed no significant evidence of unequal error variances; therefore, ordinary non-linear least-squares instead of weighted least-squares were used for parameter estimation. The data used

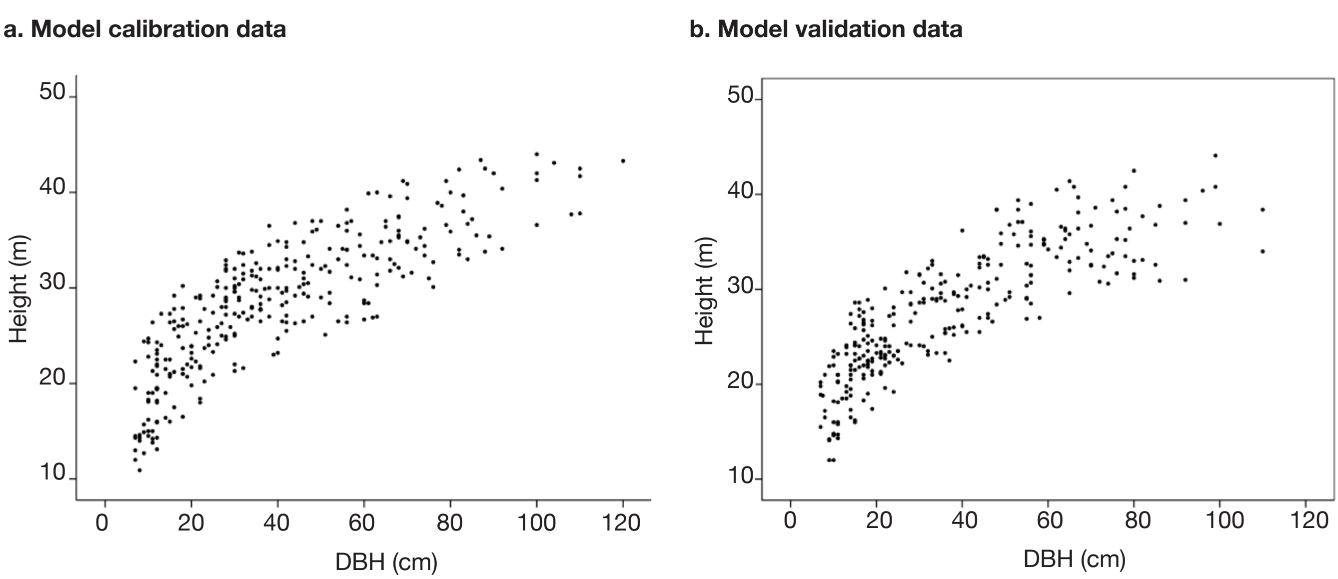

Figure 2. Scatter plot of total height (*HT*) against diameter at breast height (*DBH*) for oriental beech — *Nuage de points de hauteur totale (*HT*) en fonction du diamètre à hauteur de poitrine (*DBH*) pour le hêtre.*

Table 1. Summary statistics of diameter at breast height (*DBH*) and total tree height (*HT*) for data used in model calibration and model validation — *Résumé statistique du diamètre à hauteur de poitrine (*DBH*) et de la hauteur totale de l'arbre (*HT*) pour les données utilisées lors de la calibration et de la validation du modèle.*

	Number of trees	**DBH** (cm)				**Height** (m)			
		Mean	**Min.**	**Max.**	**STD**	**Mean**	**Min.**	**Max.**	**STD**
Model calibration	315	40.2	7.0	120.0	25.3	28.5	10.7	44.0	7.1
Model validation	290	37.6	7.0	110.0	24.1	27.7	12.0	44.1	6.7

Table 2. Non-linear height-diameter models selected for the current study — *Modèles non linéaires de la relation diamètre-hauteur sélectionnés pour l'étude.*

Model			References
Chapman-Richards:	$HT = 1.3 + a\left(1 - e^{(-b\times DBH)}\right)^{c}$	(1)	Richards, 1959; Chapman, 1961
Weibull:	$HT = 1.3 + a\left(1 - e^{\left(-b\times DBH^{c}\right)}\right)$	(2)	Yang et al., 1978
Schnute:	$HT = \left\{ 1.3^{b} + \left(a^{b} - 1.3^{b}\right)\dfrac{1 - e^{-c(DBH - DBH_0)}}{1 - e^{-c(DBH_2 - DBH_0)}} \right\}^{\frac{1}{b}}$	(3)	Schnute, 1981
Asymptotic tree height for Schnute Model:	$HT\infty = \left[\dfrac{e^{c\times D_2}\times a^{b} - e^{c\times D_1}\times h_1^{b}}{e^{c\times D_2} - e^{c\times D_1}}\right]^{\frac{1}{b}}$	(4)	Zhang, 1997
Exponential:	$HT = 1.3 + a.e^{\left(\frac{b}{DBH+c}\right)}$	(5)	Ratkowsky, 1990
Modified Logistic:	$HT = 1.3 + \dfrac{a}{\left(1 + b^{-1}.DBH^{(-c)}\right)}$	(6)	Ratkowsky et al., 1986; Huang et al., 1992
Lundqvist/Korf:	$HT = 1.3 + a.e^{\left(-b\times DBH^{c}\right)}$	(7)	Stage, 1963; Zeide, 1989

HT: tree total height — *hauteur totale de l'arbre* (m); *DBH*: tree diameter at breast height — *diamètre à hauteur de poitrine* (cm); a, b, c: model parameters to be estimated — *paramètres du modèle à estimer*; e: base of the natural logarithm — *base du logarithme naturel* (=2.71828); 1.3: a constant used to account for measuring tree *DBH* at 1.3 m above ground — *constante utilisée pour tenir compte de la mesure du* DBH *à 1,3 m au-dessus du sol*. In equation (3) — *dans l'équation (3)*: DBH_0 = diameter at 0 cm — *diamètre à 0 cm* ; DBH_2 = diameter at 150 cm — *diamètre à 150 cm*.

for modeling height-diameter relationships often contained measurements of height and *DBH* from multiple trees from the same sample plot. These groups of height-diameter measurements may violate the basic assumption of independence, as multiple observations from a single sampling unit may be highly correlated (Sharma et al., 2007). The Durbin-Watson (DW) test carried out in SigmaPlot version 12.0 (Systat Software, Inc., San Jose California USA, www.sigmaplot.comfor), was used to check the autocorrelation in non-linear models. For all of the six models, DW statistics were greater than 2, showing that there was no autocorrelation. This means that there was no violation of the independence assumption.

2.3. Model performance criteria

The selection of appropriate criteria to assess the model performance is a critical consideration. There is no single criterion for selecting the best regression model from among a number of models (Draper et al., 1998; Aertsen et al., 2010). Using multiple measurements of performance instead of single measurements is a common and more objective approach (Dawson et al., 2007; Aertsen et al., 2010). In the present study, various criteria were chosen and applied to evaluate the predictive performance of the models. In the following equations, H_i, $\ddot{H}_i$, $\bar{H}$ and p stand respectively for observed value, fitted value, mean of the observed values and number of parameters used in the model.

The most commonly used criteria to evaluate the model performance are the coefficient of determination (R^2) (Pearson, 1896) and its modification (adjusted coefficient of determination). They are estimated as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2}{\sum_{i=1}^{n}\left(H_i - \bar{H}\right)^2}$$

$$Adj.R^2 = 1 - \left(1 - R^2\right)\times\frac{n-1}{n-p-1}\cdot$$

The root mean square error (*RMSE*) is a well-accepted goodness-of-fit indicator describing the difference in observed and predicted values in the appropriate units (Harmel et al., 2007; Aertsen et al., 2010). *RMSE* is defined as follows (Aertsen et al., 2010):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2}{n}}.$$

The Akaike Information Criterion (*AIC*) is considered as one of the most reliable criteria for comparing models with a range of parameters (Burnham et al., 2002; Sharma, 2009). The model with the smallest *AIC* is considered optimal. For the least squares fit, it is calculated as follows (Dawson et al., 2007; Aertsen et al., 2010):

$$AIC = n.\ln(RMSE) + 2p.$$

The difference between the observed and the fitted tree heights is considered as the prediction error. The mean difference (*MD*) and mean absolute difference (*MAD*) are computed as follows:

$$MD = \frac{\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)}{n}$$

$$MAD = \frac{\sum_{i=1}^{n}\left|H_i - \hat{H}_i\right|}{n}.$$

Average underestimation and overestimation are indicated by positive and negative *MD* values, respectively. Mean square (*MS*) error is also considered as the indication of the model's precision. According to Zhang (1997) it is calculated as:

$$MS = MD^2 + v$$

where *MD* is the mean difference and *v* is the variance of the prediction errors. In addition to using the above-mentioned criteria, all curves generated with the different models were also checked with respect to their biological realism (Sharma, 2009). As the final step, in order to select the best model, the model validation dataset was divided into 5-cm *DBH* classes (7.5-12.5 cm, 12.6-17.5 cm, etc.) and the adequacy of the six models was evaluated on this basis. The mean prediction error computed for each *DBH* class using each of the six height-diameter models was plotted against the corresponding *DBH* class (**Figure 3**).

3. RESULTS

3.1. Model calibration

The results of the six non-linear height growth functions for oriental beech are presented in **table 3**. The coefficients of all the models were statistically significant at $a = 0.05$. **Table 4** shows the measures of performance for all six non-linear growth functions modeled in this study. Models with the lowest *RMSE* and *AIC* values and the R^2 and adjusted R^2 closest to unity are known to perform best (Aertsen et al., 2010). The adjusted R^2 values indicate that all models produced nearly identical fits explaining approximately 75% of the total variation in height. The differences in the R^2 values between the models were negligible. The *MDs* ranged from 0.001 to 0.006 whereas the *MADs* ranged from 2.894 to 2.908. In general, mean prediction errors were small for all six growth functions. Referring to **table 4**, it is evident that the differences between the model performance criteria were very small. Plotting the model residuals against the fitted values showed for all the models a random distribution of the residuals. Asymptote coefficients of the six growth functions (coefficient *a* in **table 3**), on the other hand, showed a

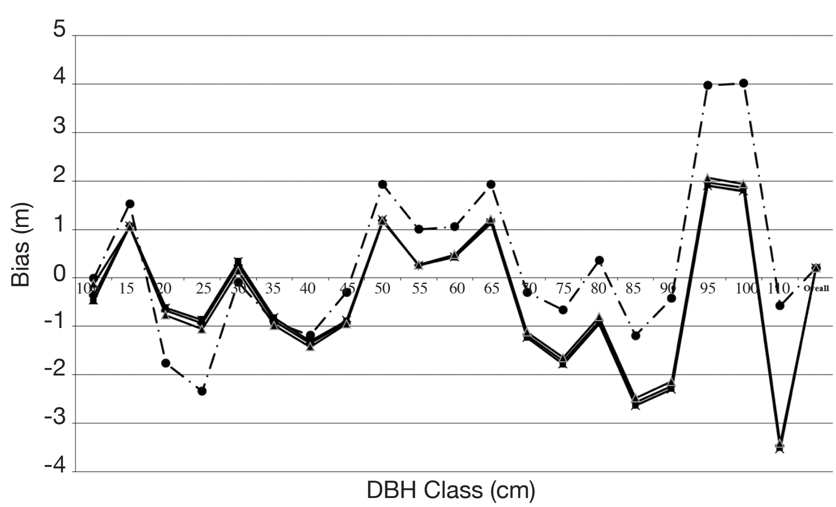

Figure 3. Average prediction error from the five tree height-diameter equations for the 5-cm *DBH* classes for beech tree species. Chapman-Richards function (■), Weibull function (♦), Schnute function (×), Modified Logistic (●) and Lundqvist/Korf function (▲) — *Erreur moyenne de prévision des cinq équations de la relation diamètre-hauteur pour les classes* DBH *des espèces de hêtre. Fonction Chapman-Richards (■), Weibull (♦), Schnute (×), fonction Exponentielle (+), fonction de Logistique Modifiée (●) et Lundqvist/Korf (▲).*

Table 3. Parameter estimates for the six non-linear height-diameter models — *Estimation des paramètres pour les six modèles non linéaires de la relation diamètre-hauteur.*

Model	Parameter	Estimate	SE
Chapman-Richards	a	45.820	5.482
	b	0.009	0.005
	c	0.399	0.041
Weibull	a	52.350	10.427
	b	0.129	0.013
	c	0.495	0.065
Schnute	a	42.333	1.557
	b	2.670	0.272
	c	0.009	0.005
Lundqvist/Korf	a	102.680	51.457
	b	3.165	0.209
	c	-0.248	0.092
Exponential	a	45.990	2.004
	b	-25.726	4.404
	c	16.194	3.790
Modified Logistic	a	65.861	15.766
	b	0.096	0.011
	c	0.568	0.098

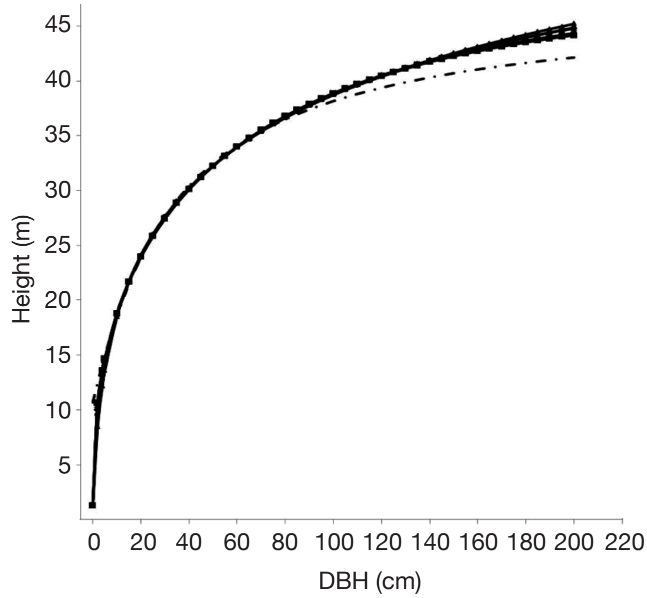

Figure 4. Simulation from the six tree height-diameter equations for oriental beech using the Chapman-Richards function (■), Weibull function (♦), Schnute function (×), Exponential function (-··-··-), Modified Logistic (●) and Lundqvist/Korf function (▲) — *Simulation des six équations de diamètre de hauteur pour les espèces de hêtre en utilisant la fonction Chapman-Richards(■), Weibull (♦), Schnute (×), la fonction Exponentielle (-··-··-), la fonction de Logistique Modifiée(●) et Lundqvist/Korf (▲).*

significant difference. The Chapman-Richards, Schnute and Exponential models showed similar asymptotic coefficients. The asymptotic coefficient of the Lundqvist/Korf function was the highest compared to the five other functions. This value was approximately twice as large as the asymptotic coefficients of the other models (except for the Modified Logistic model).

The prediction performance of all the models was investigated with tree diameters ranging from 0 to 200 cm. **Figure 4** shows the curved shapes of all six height growth models for oriental beech. For small- and medium-sized trees (*e.g. DBH* < 100 cm), the six growth functions showed similar predictions of tree height. For very small sized trees (*DBH* < 10 cm), the Exponential model predicted at zero *DBH* a height of approximately 11 m, which was much greater than the expected 1.3 m. For large-sized trees, the Lundqvist/Korf model in turn predicted remarkably greater tree heights, followed by the Modified Logistic function. The tree height estimations using the Chapman-Richards, Weibull, and Schnute functions were very similar. The Exponential function produced the smallest predictions for large-sized trees (**Figure 4**). It would be possible to confirm these results by adding a smooth line to the residual *versus* fitted graph, as this would help to show the trend. A residual distribution of the Exponential

Table 4. Performance criteria of the six non-linear height-diameter models for the calibration data — *Critères de performance des six modèles non linéaires de la relation diamètre-hauteur pour les données de calibration.*

Model	Adj. R^2	*RMSE*	*MAD*	*MD*	*AICv*	$\sqrt{v}$	*MS*
Chapman-Richards	0.7539	3.506	2.908	-0.006	401.129	3.51	12.33
Weibull	0.7550	3.498	2.899	-0.005	400.413	3.50	12.27
Schnute	0.7542	3.504	2.906	-0.005	400.944	3.51	12.31
Lundqvist/Korf	0.7564	3.488	2.886	-0.002	399.548	3.53	12.21
Exponential	0.7515	3.523	2.911	-0.001	402.664	3.53	12.45
Modified Logistic	0.7556	3.494	2.894	-0.004	400.068	3.50	12.25

function showed a slight trend towards higher positive residuals with an increased response value. The curves extracted from the Richards, Weibull, Schnute, Modified Logistic and Lundqvist/Korf functions were very similar, but the asymptotic coefficient resulting from the Lundqvist/Korf gave an unreasonable value for beech in the study area.

3.2. Model validation

The values of Adj R^2, *RMSE*, *AIC*, *MD*, *MAD*, $\sqrt{v}$ and *MS* are presented in **table 5** for each model based on the validation dataset. Referring to this table, it is clear that the Chapman-Richards model performed slightly better than the other models. The overall mean prediction errors ranged from 0.189 to 0.206 m depending on the model. The Lundqvist/Korf model had the highest *MD* value, but the differences between all the models were negligible. As mentioned above, the model validation dataset was divided into 5-cm *DBH* classes and the *MD* for each *DBH* class was calculated for each of the non-linear models. The *MD* for each *DBH* class was plotted against its corresponding *DBH* class (**Figure 3**) in order to select the best model. For small- and medium-sized trees, this plot showed similar mean prediction errors for all six growth functions, except for the Modified Logistic model. For the large-sized trees, on the other hand, all models showed higher mean prediction errors.

4. DISCUSSION

It is apparent from the model statistics that each growth function was equally fitted to the tree height-diameter data. The models accounted for approximately 75% of the total variation in height. This is consistent with the findings reported by Huang et al., 1992; Zhang, 1997; Peng et al., 2001 and Krisnawati et al., 2010. These authors also observed similar fits for all six functions. The models developed in this study explained a relatively high proportion of the total variation in observed tree height, although in some other studies (*e.g.* Zhang et al., 1996; Colbert et al., 2002; Leduc et al., 2009; Sharma, 2009; Lumbres et al., 2011) the total explained variation was much higher by comparison. In order to select the best model, several features need to be considered. We found that the predicted height at zero DBH with the Exponential model was much greater than the theoretical 1.3 m. Some researchers state that biological logics are also important and should always be considered (*e.g.* Vanclay et al., 1997; Ratkowsky, 1990; Schabenberger et al., 2002; Sharma, 2009). Therefore, the Exponential model was excluded from further analysis here. Plotting the MD for each DBH class against its corresponding DBH showed that the Modified Logistic was the worst model based on this evaluation criterion, and thus this model could also be discarded from further analysis. The Lundqvist/Korf model, on the other hand, had the highest estimates for the asymptotic value, which was also unreasonably high. This feature of the Lundqvist/Korf function has also been observed in other studies on tree height and *DBH* modeling (Zeide, 1989; Moore et al., 1996; Zhang, 1997; Peng et al., 2001; Krisnawati et al., 2010). According to Zhang (1997), the asymptotic coefficient parameter has the lowest stability in non-linear growth function modeling. Therefore, biologically unreasonable upper asymptotes may be computed by fitting these growth functions using the least squares method, especially when few data observations exist near the asymptote. Care should therefore be taken in extrapolating the models beyond the calibration data range, because overprediction or underprediction may occur for large-sized trees. To overcome this problem, some researchers have constrained the growth functions by fixing the asymptote at a constant value such as a champion big tree and estimating all other parameters in the models (*e.g.* Shifley et al., 1984; Brewer et al., 1985; Zhang, 1997). However, a champion big tree was not available in this study for oriental beech. The Chapman-Richards, Weibull, and Schnute functions all showed superior prediction performance in terms

Table 5. Performance criteria of the six non-linear height-diameter models for the validation data — *Critères de performance des six modèles non linéaires de la relation diamètre-hauteur pour les données de validation.*

	Adj. R^2	*RMSE*	*MAD*	*MD*	*AIC*	$\sqrt{v}$	*MS*
Chapman	0.7550	3.302	2.750	-0.203	352.440	3.30	10.94
Weibull	0.7548	3.304	2.751	-0.204	352.600	3.30	10.96
Schnute	0.7550	3.303	2.751	-0.202	352.480	3.30	10.95
Lundqvist/Korf	0.7541	3.310	2.757	-0.206	353.130	3.31	10.99
Exponential	0.7546	3.308	2.746	-0.189	352.890	3.31	10.98
Modified Logistic	0.7546	3.306	2.753	0.204	352.800	3.43	11.79

of mathematical features, biological interpretation of parameters and accurate prediction. Nevertheless, we recommend the Chapman-Richards model for this study area because of its slightly better predictive performance. Our results are consistent with findings reported by Zhang (1997) and Peng et al. (2001). In general, the Chapman-Richards model should be considered as the best model for modeling height growth of oriental beech throughout the study region. Since the height-diameter relationship in a forest varies due to the variability in site and stand conditions, a single height-diameter relationship may not be appropriate for estimating all the possible relationships that may be found within a forest (Krisnawati et al., 2010). There are two possible alternative approaches for estimating tree height. The first is to develop a height-diameter model separately for each stand and the second is to use generalized height-diameter models in which variability in site and stand conditions is considered by including additional stand variables as well as tree diameter (*e.g.* Bi et al., 2000; Staudhammer et al., 2000; Sánchez et al., 2003; Sharma et al., 2004; Krisnawati et al., 2010). The first approach is, however, time-consuming and costly, whereas the second is practical and could provide more accurate height estimates.

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