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Observational signatures of rapidly rotating, pulsating B stars

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Abstract: The effects of the geometric distortion of a rapidly rotating star on the relative amplitudes of radial velocity and luminosity variations are modelled. The possibility of determining the inclination angle of the rotation axis of a rapidly rotating star, from observations of its radial velocity and luminosity variations, is then explored and the nature of these effects for a model of an early B main-sequence star are presented. It is found that this approach holds sufficient promise that more detailed calculations should be embarked upon.

1 Introduction

Three phenomena contribute to the observed light variability of a star:

1. The changing direction of the unit vector normal to the stellar surface, relative to the observer;
2. The changing projected area of a light-emitting element of surface, as seen by the observer;
3. The changing surface brightness of the surface element, due to a change in its temperature.

Both pulsation and rotation contribute to the Lagrangian displacement of elements of stellar material from their positions in a spherically symmetric equilibrium configuration. Buta & Smith (1979) derived expressions for the light variability of a pulsating star in the absence of rotation. Subsequent work has explored various aspects of the observational signatures of pulsating stars that are due to rotation (see, for example, Townsend 2003). Some studies of rotational effects on stellar pulsation ignore the distortion of the stellar surface due to rotation. This is usually justified by arguing that these effects are proportional to the square of the rotation velocity Ω, and thus are second-order effects. We investigate whether the role of the inclination of the rotation axis (to the line of sight from the observer) might be identified by taking these second-order effects into account.

2 Light variations

We model the light curve for a star by superposing first order pulsational effects on a second order rotational geometry. This assumes that there is no coupling of pulsation modes with the rotation. Since our interest here is in the geometric effects of rotation, and not the temporal effects, this approximation is acceptable. The luminous power emitted at wavelength λ by an element of stellar surface is given by

$$L_0 = (F_\lambda d\vec{A}) \cdot \hat{n} \tag{1}$$

while the power detected by an observer is given by

$$L = (hF_\lambda d\vec{A}) \cdot \hat{z} \tag{2}$$

where

$$\begin{aligned}
F_\lambda &= \text{radiative flux through element at wavelength}\lambda \\
d\vec{A} &= \text{vector area of surface element, i.e. } d\vec{A} = |d\vec{A}|\hat{n} \\
\hat{n} &= \text{unit normal to surface element} \\
h &= \text{limb darkening function at wavelength } \lambda \text{ at position of surface element} \\
\hat{z} &= \text{unit vector in direction from star to observer.}
\end{aligned}$$

Buta & Smith (1979) write the Lagrangian displacement of an element on the stellar surface, due to pulsation, as

$$\delta R_{pul}(\theta, \phi, t) \;=\; a(R) Y_{lm}(\theta, \phi) e^{i\sigma t}. \tag{3}$$

This assumes that the tangential motion of the stellar material is negligible. The effect of rotation is to distort the stellar surface. We model the distortion by a Clairaut-Legendre expansion, as suggested by Tassoul (1978):

$$R \;=\; R_0 \left(1 - \sum_{n=1}^{\infty} \epsilon_{2n}(R_0) P_{2n}^0(\cos\theta)\right). \tag{4}$$

We have established that, for small distortion amplitudes, terms with n $>$ 1 can be neglected even for equatorial rotation speeds as high as 400 km/s. We may therefore express the rotational distortion with the single term

$$\delta R_{rot} = R - R_0 = -R_0 \epsilon_r P_2^0(\cos\theta). \tag{5}$$

Adding the pulsational distortion to the rotational one, we get:

$$\delta R \;=\; \delta R_{pul} + \delta R_{rot}. \tag{6}$$

In the Cowling approximation, a change in radius of the star causes a change in pressure given by

$$\frac{\delta P}{P} \;=\; \left[\frac{\ell(\ell+1)}{\omega_{\mathrm{nd}}^2} - 4 - \omega_{\mathrm{nd}}^2\right] \frac{\delta R}{R} \;=\; f_p(\ell, \omega_{\mathrm{nd}}^2)\, \frac{\delta R}{R} \tag{7}$$

where ω_{nd} denotes the dimensionless pulsation frequency. Assuming that surface elements distort adiabatically:

$$\frac{\delta T}{T} \;=\; \frac{\Gamma_2 - 1}{\Gamma_2} \frac{\delta P}{P} \;=\; \frac{\Gamma_2 - 1}{\Gamma_2} f_p(\ell, \omega_{\mathrm{nd}}^2)\, \frac{\delta R}{R} \;=\; f_T(\ell, \omega_{\mathrm{nd}}^2)\, \frac{\delta R}{R}. \tag{8}$$

In the blackbody approximation, a temperature change effects the following change in the radiative flux at wavelength λ:

$$\frac{\delta F_\lambda}{F_\lambda} = \frac{xe^x}{e^x-1}\frac{\delta T}{T} = \frac{xe^x}{e^x-1} f_T(\ell,\omega^2_{\rm nd})\frac{\delta R}{R} = f_\lambda(\ell,\omega^2_{\rm nd},T)\frac{\delta R}{R} \tag{9}$$

where $x = hc/kT$. We choose values for the arguments of f_λ that are appropriate for an early B-type p-mode pulsator. A practical reference frame, from the observer's point of view, is one that has its origin at the centre of the star and its z-axis pointing in the direction of the observer. We denote the coordinates in this frame by

$$(r,\vartheta,\varphi) \tag{10}$$

We now employ the limb-darkening law

$$h(\cos\vartheta) = \frac{6}{3-u}(1-u+u\cos\vartheta) \tag{11}$$

with

$$\cos\vartheta = \hat{n}\cdot\hat{z}. \tag{12}$$

We choose a value of u appropriate for early B stars, using the tables in Al-Naimiy (1978). The total limb-darkening function, including the distortion of the stellar surface, is given by Buta & Smith (1979).

$$h+\delta h = \frac{6}{3-u}\left[1-u+u(\hat{n}+\delta\hat{n})\cdot\hat{z}\right] = \frac{6}{3-u}\left[1-u+u\left(\frac{d\vec{A}+\delta d\vec{A}}{\left|d\vec{A}+\delta d\vec{A}\right|}\right)\cdot\hat{z}\right]. \tag{13}$$

The term in round brackets on the right hand side represents a change in direction of the surface normal due to the distortion of the surface. We can now calculate the total flux emitted by the star in the direction of the observer. We explore the utility of this approach by executing the explicit calculation of this flux for a rotationally distorted early B star pulsating in a single p-mode, as a first test case. In our calculation, we ignore all the terms which were established (algebraically) to contribute negligibly to the flux. The explicit formulae are complicated, containing large numbers of terms, and cannot be displayed here. Once we have calculated the variation in luminosity of the model star, the variation in its magnitude is readily obtained.

3 Radial velocity variations

'Radial velocity' means the velocity of the stellar surface in the direction ($\hat{z}$) pointing towards the observer. The Eulerian variation in the velocity of a given surface element of the star due to pulsation is given by

$$\vec{v}\,' = \delta\vec{v}-\vec{\xi}\cdot\nabla\vec{v} = i(\omega+m\Omega)\vec{\xi}-\Omega(\xi_\phi\sin\theta\,\hat{r}+\xi_\phi\cos\theta\,\hat{\theta}) \tag{14}$$

where $\vec{\xi}$ denotes the Lagrangian displacement of an element of the stellar surface due to pulsation. We use a second-order rotational modification for the pulsation frequency:

$$\omega \approx \omega_0-(1-C_1)m\Omega+C_2\frac{\Omega^2}{\omega_0^2} \tag{15}$$

and calculate the radial velocity projected towards the observer:

$$\overline{v}_{rad} = \int v_{obs}(\theta, \phi) \frac{dL(\theta, \phi)}{L} \tag{16}$$

where v_{obs} denotes the total velocity of a surface element towards the observer, i.e.

$$v_{obs} = \vec{v} \cdot \hat{z} \tag{17}$$

with

$$\vec{v} = \Omega r \sin\theta \, \hat{\phi} + \vec{v}'. \tag{18}$$

4 Results

We now illustrate the effects of the rotational distortion of a pulsating star's geometry on its observational signatures. The first four figures deal with radial ($\ell = 0$) and dipole sectoral ($\ell = 1, m = 0$) modes respectively. Figures 1 and 2 display our calculated ratios of radial velocity variation to variation in B magnitude, as a function of equatorial rotation speed, when the geometric distortion is *not* taken into account. The numbers appearing on the right hand side of the curves, inside the plots (0, 22.5, 45, etc.), denote the inclination angles used in the calculations. Figures 3 and 4 display the calculated ratios when the geometric distortion *is* taken into account. Figures 5 and 6 display the corresponding comparison for a quadrupole tesseral mode. The clear dependence of the ratios on the inclination angle of the rotation (and pulsation) axis only becomes apparent when the geometric distortion is included in the calculation.

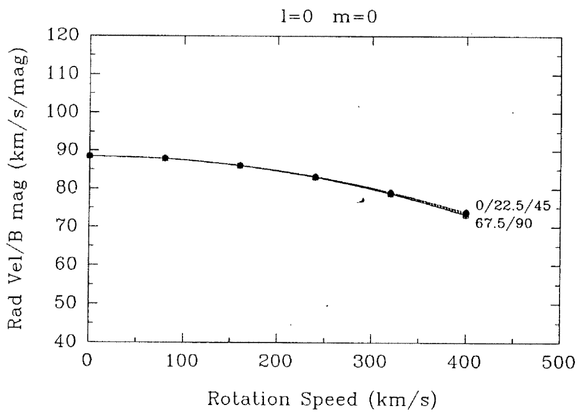

Figure 1: Amplitude ratios calculated without rotational distortion, for a radial mode.

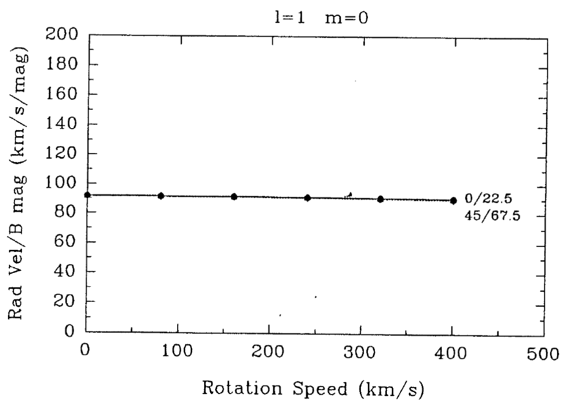

Figure 2: Amplitude ratios calculated without rotational distortion, for a dipole sectoral mode.

References

Al-Naimiy, H.M. 1978, ApSS 53, 181

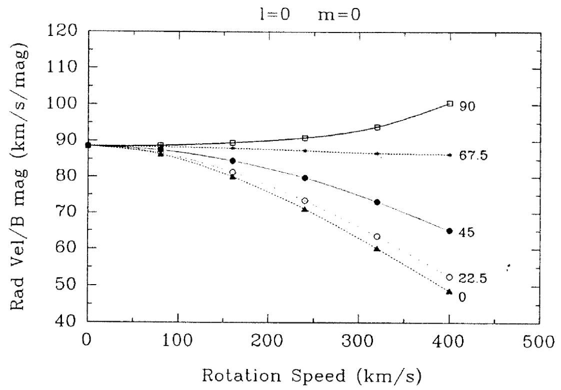

Figure 3: Amplitude ratios calculated with rotational distortion, for a radial mode.

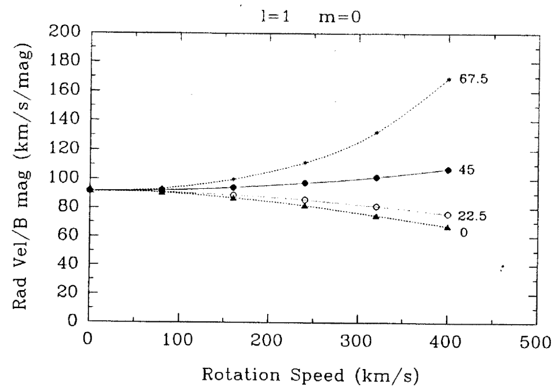

Figure 4: Amplitude ratios calculated with rotational distortion, for a dipole sectoral mode.

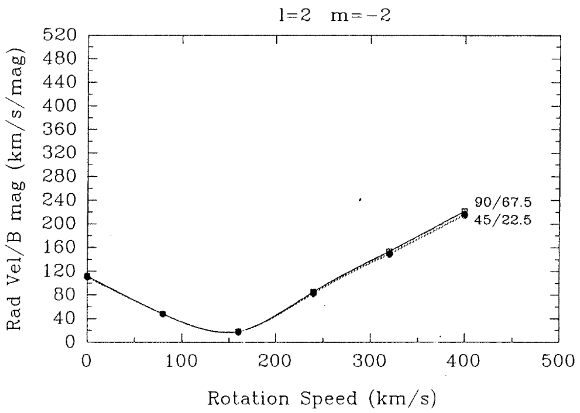

Figure 5: Amplitude ratios calculated without rotational distortion, for a quadrupole tesseral mode.

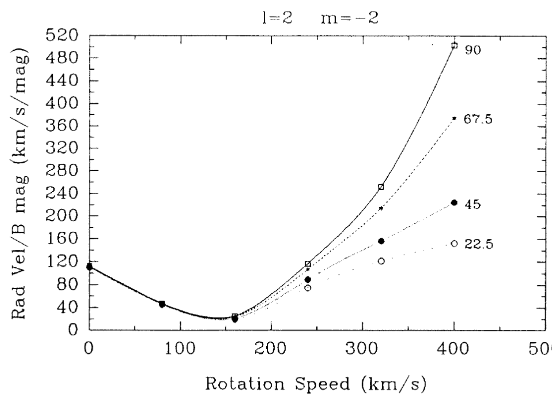

Figure 6: Amplitude ratios calculated with rotational distortion, for a quadrupole tesseral mode.

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