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Hypercyclicity criteria and the Mittag-Leffler theorem

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Dedicated to the memory of Pascal Laubin.

Abstract

We show that different hypercyclicity criteria are equivalent by using the abstract version of Mittag-Leffler theorem. We also reduce to the context of invertible operators an open problem of Herrero which asks about the hypercyclicity of the direct sum of a hypercyclic operator with itself. [1]

1 Introduction

One of the "wildest" behaviours that a linear operator $T : E \to E$ can exhibit is the existence of vectors $x \in E$ whose orbit $\text{Orb}(T, x) := \{x, Tx, T^2x, \dots\}$ is dense in E. In such a case T is called *hypercyclic* and x is a hypercyclic vector for T. This is only allowed for infinite dimensional spaces E (see, e.g., [7] and [4]). The first example was given by Birkhoff [6] who showed that the translation operator $T_a : \mathcal{H}(\mathbb{C}) \to \mathcal{H}(\mathbb{C})$, $f(z) \mapsto f(z+a)$, $(a \neq 0)$ is hypercyclic on the Fréchet space $\mathcal{H}(\mathbb{C})$ of entire functions endowed with the compact-open topology. Later, MacLane [13] proved the hypercyclicity of the differentiation operator $D : \mathcal{H}(\mathbb{C}) \to \mathcal{H}(\mathbb{C})$, $f \mapsto f'$. The first example of a hypercyclic operator on a Banach space was given by Rolewicz [15] showing that the weighted backward shift $\lambda B : l_p \to l_p$, $(x_1, x_2, \dots) \mapsto (\lambda x_2, \lambda x_3, \dots)$ is hypercyclic if $|\lambda| > 1$. All these proofs were direct, but probably the argument of Rolewicz indicated the possibility to give some kind of general criterion under which an operator is hypercyclic. This criterion was finally found by Kitai [12] in her unpublished Phd Thesis. Later, Gethner and Shapiro [8] rediscovered the criterion. Since then many hypercyclicity criteria have been given and our intention here is to unify these criteria by using a Mittag-Leffler argument.

Our framework will be continuous linear operators $T : E \to E$ $(T \in L(E))$ on $\mathcal{F}$-spaces (i.e., complete and metrizable topological vector spaces) E. The following is, essentially, the original hypercyclicity criterion (see [8, Thm. 2.2 and Remarks 2.3]):

Theorem 1.1 (Kitai/Gethner-Shapiro) *Let E be a separable $\mathcal{F}$-space and $T \in L(E)$. If there are dense subsets $X, Y \subset E$, a map $S : Y \to Y$, and an increasing sequence (n_k) of natural numbers such that*

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