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The behavior of solutions of some semilinear wave equations in one space dimension near their blow-up curve

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Dedicated to the memory of Pascal Laubin

1 Introduction and statement of the results

In this paper we shall consider the Cauchy problem

$$\Box u = F(u, u') \text{ if } x \in \mathbb{R},\ t > 0, \tag{1.1}$$

$$(\partial_t^j u)(x, 0) = \psi_j(x),\ j = 0, 1,\ \text{if } x \in \mathbb{R}, \tag{1.2}$$

where $\Box = \partial_t^2 - \partial_x^2$ is the d'Alembertian, $F \in C^1(\mathbb{R} \times \mathbb{R}^2)$, $u' = (\partial_x u, \partial_t u)$, and $\psi_j \in C^{2-j}(\mathbb{R})$, $j = 0, 1$. We shall put $\mathbb{R}^+ = \{s \in \mathbb{R}, s > 0\}, \overline{\mathbb{R}^+} = \{s \in \mathbb{R}, s \geq 0\}$. It is well known and easy to verify that one can find an open neighborhood V of $\mathbb{R} \times \{0\}$ in $\mathbb{R} \times \overline{\mathbb{R}^+}$ such that (1.1), (1.2) has a (unique) $C^2(V)$ solution. To be more precise, if $x \in \mathbb{R}$ and $t > 0$, put $K^-(x,t) = \{(y,s) \in \mathbb{R} \times \overline{\mathbb{R}^+},\ s < t,\ |x - y| < t - s\}$. If $\mathcal{U}$ is an open subset of $\mathbb{R} \times \overline{\mathbb{R}^+}$, one says that $\mathcal{U}$ is an influence domain if for any $(x,t) \in \mathcal{U}$, one has $\overline{K^-(x,t)} \subset \mathcal{U}$. Let Ω be the union of all influence domains containing $\mathbb{R} \times \{0\}$ in which (1.1), (1.2) has a (unique) C^2 solution. Then Ω is the maximal influence domain with that property. One can check that, for all $x \in \mathbb{R}$, $\{t > 0,\ \{x\} \times [0,t] \subset \Omega\} \neq \emptyset$. Put $\varphi(x) = \sup\{t > 0,\ \{x\} \times [0,t]\} \subset \Omega$. Then either $\varphi \equiv +\infty$ or φ is everywhere finite and $|\varphi(x) - \varphi(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$. In [2], [3], under suitable assumptions on the initial data, a study was made of the case that $F(z, (p, q))$ is independent of (p, q), behaves like z^r, $r > 1$, as $z \to +\infty$, and is bounded below as $z \to -\infty$. When