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MATHEMATICS IN THE REGION AACHEN-LIÈGE-MAASTRICHT FROM CAROLINGIAN TIMES TO THE 19TH CENTURY

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1. INTRODUCTION

In this lecture (*) I would like to treat a hobby of mine, namely the history of mathematics in the triangle Aachen-Liège-Maastricht, from the time of Charlemagne until the last century. Since I am not a professional historian of mathematics, I hope the presentation will not appear to lack rigor from the perspective of the expert. At the same time, since this is a general audience, I will concentrate on the mathematicians themselves, rather than on the content of their contributions. Since we are dealing with a period of about a thousand years, I must necessarily be brief and selective in presenting such an overview. I chose to concentrate on those mathematicians who were born and/or whose families originated in this region, not upon those who were active here but came from elsewhere. Since Aachen, Liège and Maastricht did not become university cities, as did Paris (in 1220), Cologne (in 1388), Rostock (1419), Louvain (1426), Trier (1457), Mainz (1467), Leyden (1575), Groningen (1614), or Utrecht (1636), it is to be expected that local mathematicians tended to leave the region in order to work in these and other university towns.

As a starting premise, I would like to suggest that a history of mathematics cannot be treated adequately in total isolation, but rather within the context of natural philosophy as a whole. This applies especially to mediaeval mathematics which cannot be studied without due attention to the history of mediaeval science, in particular the quadrivium. For this reason I will bring astronomy and music into the discussion of the earlier period.

Mathematics commonly constituted but a part of the œuvre of mediaeval scholars, their main areas of interest, usually as monks, being philosophy and theology. Since the disciplines espoused by such scholars were generally not independent of one another, the history of mathematics should, according to Murdoch-Sylla [83], be viewed not only within the broader frame of the history of mediaeval science, but also of philosophy, and even of theology. Later too, between the 15th and 18th centuries, mathematics hardly was an independent discipline, but associated with astronomy, natural philosophy, and the applied sciences, like trade, building, surveying, navigation or military engineering. Those in the teaching profession usually held professorships attached with names such as « Philosophy and Mathematics », or « Mathematics, Astronomy and Physics ».

I am fully aware of the fact that the history of any field in a specific geographical region cannot be discussed in isolation. For this purpose I will consider the history

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of mathematics in the triangle Aachen-Liège-Maastricht within the context of mathematics in the Low Countries and Germany.

In order to be able to understand and appreciate the mathematics and astronomy of Carolingian times, some idea of its antecedents and level is needed, including its transmission from Roman (or possibly Greek) sources. Familiarity can be assumed with the compendia of Cassiodorus (c. 480-c. 575), and Isidore of Sevilla (c. 560-c. 633) [87], [32] (namely his 20-book cyclopaedia *Etymologiae* and his 48-chapter *De natura rerum*, which stood for the sum of human knowledge during many centuries), with the manuals of Bede the Venerable (673-735), possibly with Martianus Capella (c. 410/39) [110] and with fragments of the Roman agrimensores (= land-surveyors, such as Sextus Julius Frontinus, Hyginus Gromaticus, Balbus, Marcus Junius Nipsus, Epaphroditus, Siculus Flaccus [33] [57]). Since Cassiodorus basically wrote up an extract of the work of his teacher Boethius (c. 481-524), not even the complete Boethius was always known. Concerning geometry, for example [59, I, p. 80), the axioms and propositions from the first four books of Euclid's *Elements* were available, but without proofs. The *arithmetica* of Boethius had some number theory derived from Nicomachus of Gerasa (c. 100) [86]. Concerning Isidore's compilation, much of it is based on Pliny's (†79) 37-volume *Naturalis historiae* [88] [89].

Apart from acquaintance at least with parts of Pliny's work (the eighth-century version at Leyden is believed to have been copied in Bede's time in a northern English scriptorium from an ancient exemplar [11, pp. 264, 290]), especially that of Vitruvius (†26 B.C.), namely his *De architectura* (the nineth of the ten volume set being devoted to astronomy), can be taken for granted, probably at the latest by 825. As a matter of fact, the oldest surviving manuscript of Vitruvius' work, the Harleianus 2767, was copied from a lost archetype most probably at Aachen [118]. Whether Ptolemy's Almagest (127-141) itself [64,94] was known at the time is most improbable, even though Charlemagne in 796-802 stood in contact with caliph Harun-ar-Raschid (766-809) of Bagdad, who is connected with a translation of Ptolemy from Greek (*) into Arabic. (The earliest translation into Latin was carried out in Sicily c. 1160.) See [64, pp. 9, 18, 20, 65]. However, scholars at the time were probably unable to understand the sources mentioned above sufficiently well to attempt strictly original work on their own. It is doubtful that they were interested in or capable of doing much more than copying passages from them.

In this connection, the treatises of Martianus, Boethius, Cassiodorus, and Isidore were mainly compilations in Latin of ancient learning, largely of Greek scholarship, and transmitted mainly through the Spanish peninsula and Britain, via Bede and the Irish monks.

2. AACHEN (795-840)

Let me now list about ten events which account for the fact that Emperor Charlemagne's court at Aachen was a center of mathematical activities at the time.

(i) The correspondence on mathematical and astronomical questions between Charlemagne and the Anglo-Saxon Alcuin (*York, ca. 730-804), since 781 director of the court school (stationed more permanently at Aachen since 794). This cor-

(*) During a Byzantine diplomatic mission to Charlemagne in 781 the Greek Elissaios remained behind in order to teach Charlemagne's daughter Rotrud Greek (see [119, p. 36]).

respondence dates from 796 onwards, when Alcuin became director of the cathedral school of St. Martin in Tours [151], [153].

(ii) The foregoing correspondence formed the foundation of (*De cursu et*) *De saltu lunae ac bissexto*, an astronomical text probably written by Alcuin. The correspondence and the text reveal that Alcuin was familiar with the work of Bede, the teacher of his teachers Egbert and Aelbert, as well as with the works of Pliny (and Vitruvius). Alcuin is most probably also the author of *Propositiones ad acuendos iuvenes* (which may be identical with *De formulis arithmeticae artis*), a text recently critically edited by M. Folkerts [39]. Both are the first books on mathematics written in Central Europe.

(iii) The Visigoth Theodulf († 821), from the Spanish Mark [72], spent the years 790 to 798, after which he became Bishop of Orléans, at the court. His manuscript *De natura rerum atque de ratione temporum libri quattuor* is in part a compilation of the works of Isidore and Bede.

(iv) The designer and builder of the cathedral at Aachen, constructed between 794 and 800, is still unidentified but is said to have been the Frank Odo (= Odilo or Eudes) of Metz according to a later source; he must be regarded as an « applied Mathematician » in view of the precise mathematical proportions and statics of the cathedral. G. Bandmann [6] feels that the cathedral was conceived and planned by Charlemagne with the assistance of the Langobard Paulus Diaconus (c. 720-79?) (and Alcuin?) around 786-787. Concerning the geometrical knowledge of master masons, see [7], [103], [117].

(v) The Franconian Einhard (c. 770-840), since 793 at the court and later responsible for the erection of the cathedral, showed definite interest in astronomy in his *Vita Caroli Magni*. He was familiar with the work of Vitruvius.

(vi) The Irish monk Dungal, since c. 787 a recluse at St. Denis, was in close contact with Charlemagne from about 800 on, trying to solve his questions on astronomy and calendar reform (tables of Easter dates). He wrote a report on the eclipse of the sun in 810.

(vii) The court text of 810 on star constellations, the fifth book of which was *De ordine ac positione stellarum in signis*, also deals with computistic problems. Parts of it are compilations of individual works of Pliny, Isidore and Bede.

(viii) The court chronicler of Emperor Louis the Pious was so interested in a description of the stars and firmament that he was called *Astronomus*. Kepler cited him in his *Astronomiae pars optica*.

(ix) The Irish monk Dicuil, since c. 800 in contact with Charlemagne, is the author of the *Mensuratio orbis* of c. 825, a work on cosmography partly based on Pliny, as well as of a text on the computus.

(x) Walahfrid Strabo (c. 808-849), regarded as the key representative of mathematics on the Reichenau, spent the years 829 to 838 at the court in Aachen as teacher of Charles the Bald (*823), son of Louis and Empress Judith. Strabo's poems reveal familiarity with Martianus [110, p. 62], [152].

Summarizing, it is my thesis that Aachen was the center of mathematical-astronomical activities in the Holy Roman Empire from 795 to 840. The court school at Aachen was at least as strong in mathematics as the later schools in Tours, Fulda under Hrabanus Maurus (780-856), the Reichenau under Strabo, Ferrières

under Lupus of Ferrières (*c. 805) and Heiric of Auxerre (*841), and Rheims under Remigius of Auxerre (*c. 841) [122]. Corbie is regarded by others ([27], [116], [121]) to be « the gromatic and geometrical capital of the mediaeval world » in the eighth century. For the literature and further details in regard to this section, the reader is referred to Butzer [18].

3. LIÈGE (995-1070)

We now come to Liège, where a key founder of what was to become the famous school of that city most probably was the Irish monk Sedulius Scotus, who spent the years c. 850 to 870 at St. Lambert Cathedral. His work was promoted by the bishops Stephan (901-920), Rather (953-956), and especially Euraklius (959-971) and Notger (971-1008), under whom Liège began to be called the « Athens of the North » [65]. Notger's student and assistant since c. 972 was Heriger of Lobbes (*c. 950-1007), known for his work on the abacus, the Roman counting table [129]. However, the greatest mathematician of tenth century Europe, and probably the most learned scholar of the time, was Gerbert of Aurillac (c. 940-1003), who became Pope Sylvester II in 999. Concerning Heriger see also [154],[155].

Concerning the state of mathematics of that time, it can be assumed that the scholars were fully aware of what had been known at Aachen almost two centures earlier. In addition, the important commentaries to Martianus Capella's 9-volume cyclopaedia by John the Scot (= Eriugena; c. 800/815-877) [128] and Remigius of Auxerre, were now available and Arabic science, derived from Spain and Sicily, was beginning to become known. Indeed, John of Gorze spent the years 954-956 at the court of Caliph Abd-er Rahman III in Cordova and vice-versa, the latter's confidant Recemund spent eigth months (955-956) in Gorze. Again, Bishop Euraklius accompanied Emperor Otto I on his 968-969 Italian campaign to Calabria. The emperor made three such campaigns and brought back about a hundred important manuscripts in 952. Gerbert of Aurillac also studied some 2 ½ years, beginning 967, at the mediaeval monastery of Vich in the Spanish Mark [72], which stood under the influence of Arabic science. A few mediaeval scholars began to read the sources in their original Greek or at least appreciate them [98]; a typical example is Bishop Rather [63] [132] a universal scholar, who also taught arithmetic [98, p. 60], was in Italy as early as 926 and spent about a dozen years as Bishop of Verona. Moreover, the mother of Emperor Otto III (himself a student of Gerbert) was the Greek princess Theophano. On the other hand, little was added that showed more familiarity with Euclid.

The points of mathematical interest that made Liège so renowed can be outlined as follows :

(i) The correspondence between Gerbert [44],[131] and Adalbold of Liège of 997-1003. This includes methods to measure the area of a circle by proceeding from the corresponding square, as well as the volume of a sphere from the cube, by taking π as equal to 22/7. Apart from these geometrical problems, Adalbold was interested in arithmetic and computation, and no doubt came to grips with problems that most of his learned contemporaries never even thought of. See U. Lindgren [73],[136]. Adalbold (*Liège ?, c. 960/970-1026), a student of Notger and Heriger, was a teacher at Lobbes from 999 to 1003 and became archdeacon at Liège in 1007, archbishop of Utrecht in 1010.

(ii) The exchange of at least eight letters between Ragimbold of Cologne, head of the cathedral school of Cologne, and Radulf of Liège of c. 1025. The subject matter was basically a proposition of Boethius : is the sum of the interior angles of a triangle equal to two right angles? One did not try to prove this result by using purely geometrical methods, but on an experimental basis, with the help of a compass or a model. Moreover, the definitions of interior or exterior angles were not clear. In connection with doubling a square, the question arose whether the value of $\sqrt{2}$ is 7/5 or 17/12. Nevertheless it was recognized but unproved that the root $\sqrt{2}$ cannot be rational. In one letter Radulf invites Ragimbold to come to see his astrolabe [78],[80]. Radulf, probably a student of Wazo (c. 980-1048; 1008-1024/32 head of cathedral school and 1041 to 1048 Bishop) and perhaps of Adalbold, had also spent some time with Bishop Fulbert (c. 960-1028; in turn a student of Gerbert) [127]; he was first designated as *magister specialis*, later as *scolarum magister* under Wazo. The correspondence was edited by Paul Tannery and l'abbé Clerval in 1901 (see [112]).

(iii) The Geometry named after Gerbert, edited by N. Bubnov [17, pp. 46-97] in 1899, probably compiled in Liège about 1025/30. This was directly influenced by the correspondence Ragimbold-Radulf; their triangle dispute was still unsolved (see [58, p. 7, 9, 11],[37],[54],[130] and the literature cited there).

(iv) An anonymous text discovered in the hospital founded by Nicholas Cusanus (1404-1464) at Cues (on the Moselle), critically edited by J. E. Hofmann [58] in 1942, finally solves the triangle dispute. The writing reveals that the text was written by a member of the Lotharingian school, the center of which was Liège, probably around 1030/35.

(v) The so-called *Geometry II of Pseudo-Boethius*, so named and edited by M. Folkerts [37] in 1970, contains a translation of axioms and propositions (without proofs) from the first four books of Euclid's *Elements*, material taken from manuals of the Roman surveyors (cf. [33],[57]), as well as a chapter on the abacus. This geometry was presumably also written in Liège since parts of its contents and the rhetorically redundant, almost stilted style, are also encountered in a text due to Franco (see below). This treatise may be placed around 1035/47, and it shows familiarity with the geometry named after Gerbert of (iii) and possibly also with the text from Cues.

(vi) The major treatise of the time was *De quadratura circuli*, a work written in six books by Franco of Liège, probably about 1047/50, first edited by C. Winterberg [123] in 1882, re-edited by A. J. E. M. Smeur [107] in 1968, and then together with M. Folkerts [40],[137] in 1976. In his attempt to perform the squaring of a circle, Franco takes a circle with diameter equal to 14, with circumference $14 \times 22/7$, namely 44, with an area $1/2 \times 7 \times 44 = 154$. The problem now is to construct the sides of a square having area 154. Not being familiar with Euclid, Franco first tried to determine the value of $\sqrt{154}$ numerically, more generally of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc. in terms of fractions. He came to the conclusion that this is not possible but gave an approximate construction, namely a recurrence relation for $\sqrt{n+1}$, given (in modern terminology) by

$$\sqrt{n+1} \cong \left\{\frac{1}{2n}\left[1 - \frac{1}{4n+2}\right] + 1\right\}\sqrt{n}.$$

The result is a good approximation to $\sqrt{n+1}$ for increasing values of n. Franco realized the problem can be solved geometrically, and did so in a single construction.

Franco also tried to solve the related problem of transforming a square into a circle, as well as to calculate the mutual excesses of a circle and a concentric, equal square. It is interesting to note that Franco's treatise was written in collaboration with Falc(h)alin, a teacher at St. Laurent. Moreover, in some of his proofs Franco cites Adalbold, Adelman, Gerbert, Razechin (= Gozechin of Mainz?, c. 1005-??), Regimbold, Wazo, Werenboldus (= Regimboldus?), and a certain Henry, but not Radulf.

All in all, Franco's treatise, the last known achievement of the Liège school, provides a wellcome addition to our knowledge of mathematics at that time. Franco, who seems to have been well acquainted with the existing mathematical literature, proceeded systematically in solving his problems. He mastered arithmetic. Although he had practically no knowledge of Greek mathematics, he nevertheless studied those problems that had intrigued the great Greek mathematicians. They were problems dealing with geometry, the irrationality of square roots, thus with problems of « pure » mathematics. Franco, possibly born in Liège (c. 1015/1020), probably was a student of Adelman (c. 1000-1061) [135], who in turn was a student of Fulbert and in 1031 Wazo's successor as head of the cathedral school. Franco became head of that school in 1066 and lived to 1083, well advanced in years.

(vii) A contemporary of Franco, the astronomer Engelbert (*), worked at the monastery of St. Laurent [134]. He was also regarded as an abacist [67].

Summarizing, it can be argued that Liège under Adalbold, Radulf, Franco and Falc(h)alin, not overlooking Wazo, Razechin or Engelbert — all of whom were probably born in or near Liège — was the focal point of mathematical studies of Western Europe during the time 995-1070. As R. Halleux [53] has rightly emphasized, it was three generations of mathematicians who built up Liège's reputation. According to C. Dawson [29], the monastic schools at Liège created a type of rudimentary university. Liège was a center of « pure » mathematics and its major achievements possibly were a result of teamwork. That these were not overwhelmingly important is of less significance than that the Liège mathematicians were some of the first mediaeval scholars to attempt problems that had fascinated the best minds of antiquity. The influence of Liège spread as far as Britain, Spain, even Poland, Bohemia and Hungary in the 11th century. Either their scholars came to study in Liège, or those from Liège may have settled in some of these countries. See J. Stiennon [111, Vol. I, pp. 77-97] and the literature cited there.

It is generally argued that Chartres [61] [62] under Fulbert, or the Reichenau [97] under abbot Berno of Prüm (c. 975-1048), Hermannus Contractus (1013-1054) and Berthold (†1088), were the mathematical centers of the time. In my view the Reichenau should be ranked second to Liège, but as a center of « applied » mathematics, its focus being on ecclesiastical chronology and astronomy. Cologne [42], under Regimbold, Wolfhelm (of Brauweiler?, c. 1015-1091) and Marianus Scottus, should probably be placed third. See also [133].

(*) Professor H. Sylvestre, Brussels, feels that Engelbert never existed. This information was kindly supplied by Monsieur Halleux.

It is curious that whereas the Meuse-Rhenish culture [99],[101],[115], reached its peak in the years 1200-1225, mathematics (*) had reached its prime in the area some 150 years earlier.

4 FROM THE MIDDLE AGES TO THE RENAISSANCE

We now move from the ninth century to 1500 in the case of Aachen, and later from the eleventh century to the sixteenth in the case of Liège, without discussing any mathematicians for the intermediate period.

Concerning mathematics as a whole, the period 1100 to 1500 was fairly quiet, the main mathematicians being Leonardo of Pisa (c. 1170-c. 1250), Jordanus Nemorarius (c. 1220), Johannes Campanus de Novara (c. 1260), Thomas Bradwardine (*Chichester, c. 1290-1349), Nicole d'Oresme (*Caen, c. 1323-1382) — perhaps the most prolific mathematician of the middle ages, as well as Regiomontanus (*Königsberg, Franconia, 1436-1476) and Nicolas Chuquet (*Paris, † c. 1500). From the southern Low Countries one can also mention such scholars as William of St Thierry (*Liège, c. 1080-1149) [49],[45], Hézelon of Liège († c. 1120) [111], Alan of Lille (= Alanus ab Insulis, A. de Ryssel; c. 1128-1202) [45], the « universal doctor » and author of *De planctu naturae*, Gerard of Brussels (c. 1187-1260) [25], the founder of kinematics in the West, Thomas of Cantimpré (*Sint Pieters-Leeuw, 1201-1261/80), the author of *Libre de natura rerum* [23],[10, Vol. 25, pp. 29 ff.], the geometer and astronomer Aegidius (Giles) of Lessines (c. 1205-1280), William of Moerbeke (= Flemingus, c. 1215-c. 1281), Siger of Brabant (c. 1235-1282) [74, pp. 171-206], Simon de Couvin of Liège, the author of *De iudicio solis in coniviis saturni* (written in 1350) [114, Vol. III, p. 305, 311], Petrus de Alliaco (= Pierre d'Ailly, *Compiègne c. 1350-c. 1420), the astronomer Johannes de Lineriis (= Jean de Linières, *Amiens ?, c. 1300-1350/55). Finally, Nicolas Cusanus of Cues is to be recalled.

The first Renaissance mathematician to be discussed here is

PETER GYMNICH (= GEYMMENICH ?, PETRUS AQUENSIS) (*Aachen, c. 1470- ?)

This humanist studied 1483-1485 under Alexander Hegius (*Ahaus, c. 1433-1498); then, in Deventer, in April of 1485 we find him under Rudolf Agricola (*Groningen, c. 1433-1485), and in 1498 he was studying law in Cologne. No later than 1503 he became a canon at the St. Martini monastery in Münster, Westphalia. Gymnich was still alive in 1523, judging by an anonymous dedication to him in his report concerning the famous religious dialogue held at Groningen in that year. Aquensis, who is designated there as *vir eruditissimus* is highly honoured as a patron of *bonae litterae*.

(*) Just as mathematics and astronomy were inseparably related during this period, so were mathematics and music. Arithmetic, music, geometry and astronomy were the four subjects making up the *quadrivium*. Now there were many musicians in this region in the middle ages, such as Rupert of Liège and Cologne (c. 1074-1129), Jacobus of Liège (c. 1260-1330) and Johannes of Valkenburg (active c. 1300). I find one of them especially important, namely Johannes Ciconia (= Ciwagne, Cicogne), born in Liège c. 1335 and died in Padua in 1411 (?). In fact, the period 1400-1440 in musical history is called the epoch of Ciconia. In this regard one may consult the works of A. Auda [4], J. Smits van Waesberghe [108], H. Besseler [8], H. Hüschen [60], S. Clercx-Lejeune [26], and J. Maillard [75].

Although he is reported to have written a number of works, none seem to have survived. However, Gymnich wrote an introduction to Timann Kemner's (*Werne, 1470-1535) book *Compendium etymologiae et syntacticae grammaticae* (Jacobus van Breda, Deventer 1502; Quentell, Cologne, ²1513). This five-page dedication, a German translation of which appears in [19], is a typical humanistic essay of the time. It does reveal that Gymnich had mathematical interests.

Concerning his actual role as a mathematician, very little is known. The historian Hermann Hamelmann (*Osnabrück, 1526-1595) calls him *Magnus fuit mathematicus*. Johannes Longicampianus (*Langenfeld?, c. 1490/95-1529), Professor of Mathematics at the University of Wittenberg, is reported to have said in his lectures of the late winter of 1529 that, in his extensive trips through all of Germany and beyond, he had not met any better mathematicians than Peter Gymnich of Aachen (in Münster) and Bartholomaeus of Cologne (in Minden).

For further details and the corresponding literature, see [19].

Dietrich Tzwyvel (= Theodor Zwivel) (*Zweifall, 12 km southeast of Aachen; c. 1480-1543)

Nothing definite is recorded about the colleges or universities where Tzwyvel studied. At the latest from 1512 on, or perhaps as early as 1505, he lived in Münster, Westphalia. He is best known for his activities as a book-printer and music-theorist, perhaps because these have been examined in detail (cf. J. Prinz [92]; K. G. Fellerer [35]). At least three of his books deserve mention here :

Opuscula duo de numerorum praxiquae algorithmi dicuntur (Quentell, Cologne 1505); *Ars supputatoria calcularis* (Quentell, Cologne 1508); *Elucidarium computi ecclesiastici* (Tzwyvel, Münster 1515/16).

The first is an introduction to mathematics for students of music, the second an elementary arithmetic, and the third was used in connection with university lectures in astronomy. In his monographs he cites St. Augustin, Isidore of Sevilla, Guido of Arezzo (*990), Wilhelm of Hirsau (*1026), Bernhard of Clairvaux (*1090), and especially Boethius. Tzwyvel regarded arithmetic as the foundation of music. According to M. Cantor [24, p. 419], Tzwyvel is a German forerumer of Henricus Grammateus (c. 1490-1525). However, no historian of mathematics seems to have studied Tzwyvel's work seriously. Only then would it be possible to rank him among his contemporaries. See [19].

The question may well be raised whether the monastery schools along the Wallon-German/Dutch linguistic boundary produced mathematicians of any note. In this respect possibly Prüm, but certainly Stavelot-Malmedy [5], Kornelimünster (south of Aachen), Val-Dieu (= Gottestal, near Aubel), and Rolduc (= Klosterrath, near Kerkrade) [69],[141] are at issue. In the case of Prüm [93],[85] there are at least Regino of Prüm (†915) and Berno (of Reichenau, †1048); in the case of the other monasteries the problem does not yet seem to have been investigated.

5. The years 1525-1700; a forerumer of the calculus

This period was indeed a golden one for the Low Countries. The more southerly part produced such mathematicians and astronomers as Jean Taisnier (*Ath, 1508-1562?) [46], Jean Stadius (*Loenhout, 1527-1579), Simon Stevin (*Bruges, 1548-1620), Joannes Storms (= Sturmius; *Mecheln, 1559-1650), Adriaan van Roomen

(*Louvain, 1561-1615) [139],[13],[138], Godfried Wendelen (*Herk-de-Stad, 1580-1667) [104], Charles Malapert (= Malapertius; *Mons, 1581-1630), Gregorius van St. Vincentius (*Bruges, 1584-1667), Arnold de Lens (*Belœil, 1590-?), Jean Charles della Faille (*Antwerp, 1597-1652), Johann Ciermans (*'s-Hertogenbosch, 1602-1648) [149], Willem Boelmans (*Maastricht, 1603-1638), Andreas Tacquet (*Antwerp, 1612-1654), Gerard van Gutschoven (*Louvain/St.-Trond, 1615-1668) [13], René-François de Sluse (*Visé, 1622-1685), Ferdinand Verbiest (*Pittem, 1623-1688), Gilles-François de Gottignies (*Brussels, 1630-1689), and Jacques-François Le Poivre (*Mons, ?-1710) [48]. Of these, Malapert, van St. Vincentius, Ciermans, Boelmans, della Faille, Tacquet, Verbiest and de Gottignies were Jesuit priests [150].

Return to Liège, the Brethren of the Common Life, a religious organization founded by Gerhard Groote (*Deventer, 1340-1384), opened their school St. Jerôme at Liège in the 15th century [91],[28],[81]. One of their members was Arnoldus Einatensis (*Eynatten?, near Aachen), the teacher of Johannes Sturm (*Schleiden, 1507-1589), the renowned humanistic educator and one of the three founders of the famous Protestant Gymnasium at Strasbourg [66],[51]. A little later the prince-bishops of Liège, especially Robert of Berghes († c. 1565) and Ernest of Bavaria (1554-1612), made various attempts to set up a university in Liège, without success. Nevertheless in 1613 the English Jesuits set up a College [31] there, specializing in physics, mathematics and astronomy. It was comparable to that of Louvain and Antwerp at the time. One of the college's first, well-known professors of mathematics and Hebrew was Franciscus Linus (actually Line or Hall, *London or Buckingham, 1595-1675) [109, Vol. 4, pp. 1840-1842; 70, pp. 71-75],[145],[148]. Our first mathematician of the period lived a little earlier. As a matter of fact he came from Maastricht (see Section 9 below). He is

Willem Raets
(*Maastricht, c. 1540-c. 1567/76)

Although little is known about Raets' life, he spent a good part of it at Antwerp, where he had made friendship with Michel Coignet (= Cognet; *Antwerp, 1549-1623), a mathematician reputed for the applications of mathematics to the sciences, and to navigation [41] in particular. Raets (*) is the author of ([10, Vol. 18, p. 583])

Arithmetica. Die fundamenten seer gronelyck verclaert, ende met veel schoonder question geillustreert, tot orbaer allen Cooplieden ende anderen handelers (Antwerp, Gillis van Diest, 1566; 8° in 112 ff.). A second book entitled *Practycke om lichtelyck te leeren visieren alle vaten metter wisselroede*, appeared in 1567 (8° in 32 ff.).

Coignet arranged for the publication of a second edition, printed 1580 in Antwerp by Hendrick Hendriesen (8° in 124 ff.), as well as for a third edition, printed 1597 in Antwerp by Jérôme Verdussen. For a short description of the text the reader is referred to [10].

Coignet, in the preface of the second edition, writes that he still regrets the passing of his very good friend Raets, who had died suddenly. Since Coignet received a ten-year royal privilege for this edition dated May 22, 1576 in Brussels, this sug-

(*) Raets may have been a descendant of the ancient aristrocatic family Ratz (= Raets, Raedts, Raitz von Frentz) of Cologne (related to the illustrious family Beissel von Gymnich), which celebrated its 1.000th anniversary in Cologne on July 15, 1900. Offshoots of the Ratzes settled in the Low Countries in the late Middle Ages, and include Le Ratz de Lanthenée (see below).

gests that Raets may have died between 1567 and 1576. In that case, Raets and Coignet, who was 20 years old in 1569, could have known each other for at most seven years.

On the other hand, A. J. M. Smeur [105, p. 36] reports that a certain Willem Raets was a juryman at Maastricht between the years 1517 and 1550. Assuming that he obtained this position at the age of 25, he would have been born in 1492, if not earlier. Since the juryman Raets would have been 77 in 1559, say, and the mathematician Raets is reported to have died in the prime of his life, H. L. V. De Groote [30],[140] feels that the mathematician was not identical with the juryman. Perhaps the juryman was the grandfather of the mathematician since christian names were usually passed on to grandsons at the time.

Our second mathematician, however, made his reputation in Liège.

Gille (Aegidius) Guillon
(*Liège, c. 1575-c. 1618)

Pastor of Sainte-Marguerite in the outskirts of Liège in 1604, Guillon spent the years 1607 to 1610 in Rome, where he made friends with the Jesuit Christopher Clavius (*Bamberg, 1537-1612), one of the outstanding teachers of mathematics of the 16th century. Upon his return to Liège he received the benefice of Saint-Martin college, and later became doyen of the collegiate chapter in Gransée (Burgundy).

Two of his works on mathematics deserve citation :

Institution de l'arithmétique avec les gettons et la croye (L. Streel, Liège 1604, 8° de 7 ff., 238 pp.).

L'algèbra de Christophe Clavius, sommairement recueillie, et traduite du Latin, par Gille Guillon, prêtre liègeois du collège de Saint-Martin. Enrichie d'un avant-propos, outre plus d'une amplification de l'algèbra, etc. (L. Streel, Liège 1612, 4° de 8 ff., 232 pp., plus 5 ff. de table de 2 ff. contenant des poésies latines).

My references are [95, p. 128], [10, Vol. 8, p. 547], [70a, pp. 39-45], [55, p. 358].

Libertus Fromondus (= Libert froidmont)
(*Haccourt, 3/6.09.1587 - 27.10.1653)

Beginning his education at Maastricht, Fromondus studied at the Collège du Faucon of Louvain, spent three years at St. Michael of Antwerp, and returned to Louvain where he taught rhetoric and philosophy for some eighteen years. He received the doctorate in theology in 1628, and in 1637 became Professor of Theology at Louvain, to succeed his friend Cornelius Jansenius (1585-1638). He is noted for the fact that he looked after the publication of Jansen's controversial book « Augustinus » in 1640, and corresponded with René Descartes, who lived in the Netherlands from 1629 to 1649.

He was the author of many books, including : *De cometa anni M.C.C.XVIII. Dissertationes Thomae Fieni et Liberti Fromondi* (*Antwerp 1619, 8°*); *Meteorologicorum libri VI* (*Plantin, Antwerp 1627, 4°*); *Anti-Aristarchus, sive de orbe terrae immobilis liber unicus, in quo decretum S. congregationis S.R.E. cardinal. An. 1616 adversus Pythagorico-Copernicanos editum defenditur* (*Antwerp 1631, 4°*); *Labyrinthus, sive de compositione continui liber unus, Philosophis, Mathematicis, Theologis utilis et jucundus* (Plantin, Antwerp 1631).

It is interesting that Fromondus' collected works were given out in Paris in 1670 and again 1709 in Rouen. His rating from a modern point of view remains open. For references one may consult A. Quetelet [95, p. 222], [2, Vol. 8, p. 145], [90, Vol. 1, p. 811-812], [10, Vol. 8, pp. 312-317], [70a, p. 77 ff.], [143, 144].

Willem Boelmans
(*Maastricht, 7.10.1603-20.10.1638)

After spending seven years in the Jesuit school at Maastricht, Boelmans joined the Society of Jesus in Mecheln in 1617. In 1620 we find him as a student in Antwerp, and 1621-1624 under the renowned Gregorius van St. Vincentius, Professor of Mathematics at Louvain (1520 to 1525). After teaching at the Jesuit gymnasia in Maastricht (1624-1627) and Gent (1627-1629), he studied theology at Louvain, and was ordained in 1632. He was Professor of Mathematics at Maximum College in Louvain from 1633 until his early death; see [20].

Of particular interest in his life is the fact that on 8.-9. August 1634 he presided over a public defense of theses of his held by four of his students, namely Johannes Groll, Phillip Jacobi (1612-1678) [150], Laurent van Schoone (1613-1677) [109, Vol. 7, p. 864], and Andreas Tacquet. This was a general tradition of those days. The programme, which functioned as the abstract of a modern colloquium, was : *Theses mathematicae, geometricae, arithmeticae, opticae, catoptricae, dioptricae, musicae, architectonicae, stereo-staticae, hydrostaticae.*

Concerning the participants, it is recorded that at least Gregorius, as well as the reputable humanist Erycius Puteanus (1574-1646) [82], were present. Tacquet became almost as well-known a mathematician as his « mathematical grandfather » Gregorius.

One part of the optical theses, that on refraction, has now been edited by A. Ziggelaar [126], who concluded that Boelmans discovered a principle equivalent to that of Fermat, and formulated the law of refraction as a consequence. Fermat only published the refraction law as the sine law in 1637, and his « principle of Fermat » as late as 1662, also deriving the sine law (already known to Willebrord Snell (1581-1626)) as an application.

Apart from the programme of the debate, an unpublished manuscript by Boelmans of 1625 is included in the literary remains of Gregorius located in the Bibliothèque Albert I at Brussels. See [20], [142]. It may be worthwhile to edit the complete programme as well as to study the latter manuscript.

René-François de Sluse (Slusius)
(*Visé, 2.07.1622-19.03.1685)

He was the son of Renard de Sluse, notary and clerk of the court at Visé, and Catherine Plorar (called Walteri). Both families had produced high civic and religious officials for some two centuries in Visé. After attending the Jesuit College (?) at Liège (or Maastricht?), Sluse spent the years 1638-1642 at Louvain (under Ciermans?, who conducted three « conferences » on mathematical sciences in 1639-1640 [150]), then eleven years in Rome and Florence studying Greek, oriental languages, mathematics, astronomy, physics, and medicine, obtaining his doctorate in law in 1643. In Italy he had made friends with the well-known mathematicians Michel Angelo Ricci (1619-1682) and Stefano degli Angeli (from Venice). In 1651 he

was appointed canon of St. Lambert cathedral at Liège, beginning his residence there in 1653, and in 1655 became head of the cathedral chapter.

Besides writing several papers, Sluse is the author of the book *Mesolabum* (J. F. van Milst 1659, enlarged edition by W. H. Streel 1668, both Liège). It is concerned, for example, with the construction of tangents to curves, of the roots of equations, the determination of the geometrical means of conic sections, with infinitesimal questions, and with the first proof of the well-known inequality later named after Jac. Bernoulli.

This book exerted a considerable influence upon the development of mathematics at that time. James Gregory (*1638) and John Wallis (*1616) praised it; Leibniz (*1646) studied it extensively during his stay in Paris (1672-1676). Sluse also stood in intensive correspondence with Christiaan Huygens (*1629), Blaise Pascal (*1623), Henry Oldenburg (*c. 1615), Wallis etc., see C. Le Paige [70b], A. R. Hall-M. B. Hall [52]. His contacts with Britain were so close that he was elected a member of the Royal Society in 1674.

All in all, Sluse was one of the most important links in the chain leading to Gregory, Leibniz and Isaac Barrow (*1630). It was the final stage before the discovery of the actual infinitesimal calculus by Leibniz, Newton (and Gregory). Sluse was no doubt one of the best Belgian mathematicians of all times. However, a final assessment of Sluse's contributions from today's standards remains to be made. It is to be hoped that the mathematicians, historians of mathematics and the professional historians of Liège will commemorate the 300th anniversary of his death on 19 March 1985, using the intervening time to especially study his many unpublished works located at the Bibliothèque Nationale in Paris. See also [36a,b] and Section 9.

6. THE EIGHTEENTH CENTURY

The 18th century was less important in mathematical terms for the Low Countries, for the Rhinelands, or Germany in general : it was an interlude, between the « Century of Genius » and the « Golden Age » of mathematics. In the Low Countries we have the commander Charles-François le Prud'homme d'Hailly, Viscount of Nieuport (1746-1827) [14], born in Paris of parents from Gand. In Germany there is Christian von Wolff (*Breslau, 1679-1754) (who was offered a professorship at the University of Groningen in 1740) and Johann Friedrich Pfaff (*Stuttgart, 1765-1825), if one disregards Gauss, Möbius and von Staudt, who only began their carreers in the 19th century.

Before discussing the 18th century mathematicians of the region, three physical scientists deserve brief mention (*). Firstly, Robert, chevalier de Lo-Looz (*District of Liège, c. 1730-1786) [10, Vol. 12, pp. 324-325], who served as colonel under the Swedes and later the French, was interested in astronomy, physics, military engineering and tactics. His collected works appeared in two volumes in London and

(*) Three other scientists, of the time of the French Revolution and Napoleon, also deserve mention here. They were among the many *Liègois*, such as the opera composer André Grétry (1741-1813), who made careers in Paris at the time : physicist, aeronaut and great inventer Étienne-Gaspard Robert(son) (*Liège, 1763-1837) [10, Vol. 19, pp. 496-507]; the physician and natural scientist Pierre-Hubert Nysten (*Liège/Beek, 1771-1818) [10, Vol. 15, pp. 46 ff.]; and the mathematician and technologist Gérard-Joseph Christian (*Verviers, 1776-1832) [10, Vol. 4, p. 97].

Paris in 1788. Further, Jan Pieter Minckelers (*Maastricht, 1748-1824), Professor at the Collège du Fauçon in Liège (1772-1778) and since 1797 at the École centrale in Maastricht, member of the Académie royale de Bruxelles, is regarded as one of the discoverers of gas-lighting and the balloon [96]. Finally, Antoine Chaudoir (*Spa, 1749-1824) became Professor of Philosophy, Astronomy and Experimental Physics at the University of Franeker (1585-1811) in Friesland [95, p. 311], [43]. We turn now to the mathematicians of the district, although they have received scant attention in treatises on the history of mathematics.

Jean-François and Jean-François(-Sébastien) Le Ratz de Lanthenée (*Thuin, 7.12.1679-1756, 24.09.1708-1778)

Although this unique pair — father and son doing mathematics together — was not born in our « triangle », since Thuin is 15 kms. southwest of Charleroi (it nevertheless belonged to the Principality of Liège at the time), they stemmed from the Rhenish family Razo (Ratzo, Ratz) of Cologne, which dates back to the tenth century. One of their many famous ancestors seems to have been Rutgher Raets (†1369), marshal of France. In the 15th century a branch of the family Ratz had settled in Cour-sur-Heure (near Thuin), by 1520 it carried the name Le Ratz, and by 1651 the name Le Ratz « seigneur de Lanthenée et St. Pierre », and lived in the castle « la Pasture » near Thuin. They intermarried with such noble families as the Helspighel and Eynatten de Thys of St. Truiden, and the Polchet of Namur [146].

Both father and son studied at the Collège de l'Oratoire in Thuin as well as at the University of Paris. Whereas the father returned to Thuin, the son lived from 1733 on with Voltaire, from 1746 with the Palatine Princes in Zweibrücken, and after c. 1750 as a courtier in Versailles. From the correspondence between father and son one can note that all but the last of the publications cited below basically were joint work. The father, who had built up a very selective library, seems to have had the main ideas; the son, who had had more formal training as a mathematician, and actually was regarded by his contemporaries as the mathematician of the family, carried them out. Concerning the contents of the publications the reader is referred to Boulmont [15].

Éléments de Géométrie (Paris 1738, 12°, 260 pp.); *Lettres à M. Voltaire sur son écrit intitulé : Réponse aux objections contre la philosophie de Newton* (Paris 1739, 8°); *Examen et réfutation de quelques opinions sur les causes de la réflexion et de la réfraction, répandues dans l'ouvrage de M. de Banières contra la philosophie de Newton, avec un essai sur l'impulsion appliquée aux phénomènes de la lumière, et quelques autres attribués à l'attraction* (Paris 1740, 8°, 50 pp.); *Nouveaux essais de physique* (Paris 1750, 12°, 109 pp.); *Essai sur une méthode de rendre les aréomètres ou pèseliquers comparables* (Paris 1769, 12°).

This material is taken from Ad. Quetelet [95, p. 279], J.-C. Poggendorf [90, Vol. I, p. 1374] and [15].

Johann Joseph Crümmel (*Aachen, 19.06.1729-12.12.1807)

Was the son of Joannes Crümmel (*1682) and Catharina Butgens (Büttgens). The Crümmels probably derive from the old aristrocratic family Crümmel of Eynatten (Merols branch), dating back to the 14th century. He was a pupil at Aachen's Jesuit Gymnasium, and later became a noted banker. Since 1752 he was a member

of the *Kunst-Rechnungsliebende Societät* at Hamburg, one of the oldest of mathematical societies.

He is the author of *Compendiöser neuer gregorianisch-ökonomisch-astrophysikalisch-geographischer Kauf- und Handels-, wie auch Planeten- und am Ende dieses Saeculi auslaufender Circular-Calender* (Aachen, 1749), and *Zum Nutzen der Algebra in allen Wissenschaften, besonders in der Handlung* (Aachen, 1756). It is interesting to note that the latter was translated into Dutch by Jacob Oostwoud and published in the Netherlands under the title *Net Nut der Algebra in alle Wetenschaapen* (Pieter Joordan, Purmerend 1776, 8°, 36 pp.); see D. Bierens de Haan [9, p. 66]. Oostwoud (1714-1784) was noted for the fact that he was the publisher of what might be called the first Dutch journal of mathematics; see P. P. Bockstaele [14].

7. THE 19TH CENTURY

This century ushers in a new course of events. The University of Liège was founded in 1817, opening up a variety of opportunities for the region [147]. The first professor of mathematics to be appointed at Liège was Jean Michel Vanderheyden (*Maaseik, 1767-1836), who had already taught in the *Faculté des Sciences* founded 1811 there under the French administration. A number of the professors of mathematics at Liège were actually born in Liège, including Joseph-Remi-Léopold Delbœuf (1831-1896), Louis-Arnold-Joseph Graindorge (1843-1896), Constantin-Marie-Michel-Hubert-Jérôme Le Paige (1852-1929) [47], Jacques-Joseph-Émile Ronkar (1857-1902) and François Deruyts (1864-1902). Also such well-known mathematicians as Eugène-Charles Catalan taught at Liège (from 1865 to 1884), the famous Italian Ernesto Cesàro (*1859) being a student of Catalan (see [47]), while the celebrated math.-physicist Joseph Plateau (*Brussel, 1801-1883) was one of the seven to receive a doctorate at Liège between 1817 and 1830 [14]. The Société royale des Sciences de Liège, founded in 1837, has been publishing its *Mémoires* since 1843 and its *Bulletin* since 1932, a good part of the articles being devoted to mathematics. Then in 1870 the Aachen Polytechnical Institute was established; some of the many mathematicians who helped build up its great reputation include Karl Hattendorf (from 1870 to 1882 in Aachen), August Ritter (1870 to 1899, in mechanics), Hermann Stahl (1882 to 1885), and Hans von Mangoldt (1886 to 1904).

However it is not my aim to consider these mathematicians, who have been discussed in expert fashion for Belgium in [34], [47], and [3], and for Aachen in [1, 100], but three other scholars. All three have at least two facts in common : they were born in Germany within 30 km from Aachen, their paternal grandfathers were born in eastern Belgium (in Verviers and Liège), and one was the teacher of the other, who in turn was a friend of the third. They are Lejeune Dirichlet, Reuleaux and Christoffel.

JOHANN PETER GUSTAV LEJEUNE DIRICHLET
(*Düren, 13.02.1805 - 5.05.1859)

Dirichlet was the seventh and last child of Johann Arnold Lejeune Dirichlet, a postmaster, salesman and town counciller, and Anna Elisabeth, née Lindner (*Aachen, 1768). His grand-father was Antoine Lejeune Dirichlet (*Verviers, 1711), a cloth manufacturer, who had settled in Düren, married Anna Koenen, and had become mayor of Düren. The former's father and grandfather both had the same Christian names, namely Remacle Antoine. To distinguish the two, Dirichlet's

great-grandfather (*Verviers, 1674) was the first to carry the name Lejeune Dirichlet. His family name was actually Derichelette, or « de Richelette » a generation earlier, a name derived from the hamlet « Richelette » located about ten km southeast of Visé (near the Walloon-Dutch language boundary). Indeed, a Thiry Lowys (= Derrick Louis) de Richelette had left Richelette to settle in Verviers by 1575. He was the ancestor of the Dirichlet family.

On the maternal side, Dirichlet's grandfather Carl Gottlieb Lintener was born in Annaberg, Saxony, but his maternal grandmother Maria Gertrud Hachtmann again came from a Düren family. This disproves the opinion expressed in practically every encyclopedia or book on the history of mathematics, that Dirichlet originated from a French emigrant or even a Huguenot family. On his father's side he came from Catholic families, on his mother's partly from Protestant ones. For the detailed genealogy, traced back in part to the 15th generation to a knight Renard de Waimes of Roanne (near Stavelot) living c. 1280, see [22a].

Dirichlet, who had the fortune to have had the great physicist Georg Simon Ohm as his mathematics teacher at the Catholic Gymnasium at Cologne, studied from 1822 to 1826 at Paris, the world center of mathematics at the time. There he made friends with Lacroix, Poisson and Fourier. Through Fourier he became interested in the form of analysis later named after him. Besides attending lectures, Dirichlet devoted himself to reading and rereading Gauss's *Disquisitiones Arithmeticae* (1801). He was probably the first to master this difficult work. The two great influences of his life were two antipodes : Fourier with his theory of heat, and Gauss with his arithmetic (number theory in present-day sense), thus both the very « applied » and the very « pure » mathematics.

Soon after his return to Germany, Dirichlet received an honorary doctorate at Bonn, spent the years 1827-1829 at Breslau and 1829-1855 as Professor at the University of Berlin, when he accepted the invitation to succeed Gauss at Göttingen.

Dirichlet wrote some fifty papers, to be found in his collected works : *G. Lejeune Dirichlet's Werke. Edited by L. Kronecker, continued by L. Fuchs, G. Reimer, Berlin 1889-1897, 2 Vols. (Chelsea reprint 1969).* Although he did not publish any books, his lectures on number theory were given out by R. Dedekind (Vieweg, in 1863, ^{4}1891; Chelsea 1968), on definite integrals by G. F. Meyer (Teubner 1871), on potential theory by F. Grube (Teubner 1876, ^{2}1887), on single and multiple integrals by C. F. G. Arendt (Vieweg 1904).

Dirichlet's most profound research is that connected with number theory. In 1837 he proved that any arithmetic progression a, $a + d$, $a + 2d$, ... in which a and d have no common factor, contains infinitely many primes. For the proof he used several novel ideas, including his *Dirichlet series*. In 1838-1842 he proved the « class number formula », conjectured by Jacobi, using integration and infinite series. Indeed, Dirichlet is regarded as the father of analytic number theory. He used his « drawer principle » to clarify the structure of unit groups and algebraic number fields. His work is the genesis of Dedekind's ideal theory. But perhaps Dirichlet's most popular work is that in analysis. He was one of the first to define and use the term « function » in its modern sense, to give a rigorous proof, under reasonable conditions — the *Dirichlet conditions* —, of the validity of Fourier series expansions, as well as to deal in potential theory with the *Dirichlet principle* and the associated *Dirichlet integral.* In view of the latter work and that on mechanics and fluid dynamics, he is also regarded as the father of mathematical physics in Germany, together with Franz Neumann (1798-1895).

Among his many famous students are Riemann, Heine, Philipp von Seidel, Eisenstein, Kronecker, Christoffel, Dedekind, Lipschitz and Bjerknes. Dirichlet is also regarded as the initiator of university lectures in mathematics as they are still held in Germany today, as well as of a type of mathematical seminar. In this regard L. C. Young [125, p. 184] states « There I now finally come to the two great *teachers*, the German equivalent of Carnot and Monge, the real founders of the German school of mathematics, Jacobi and Dirichlet ». All in all, Dirichlet probably belongs to the first 35 best mathematicians of all countries who were active until 1860. For further particulars one may consult Butzer-Jansen-Zilles [22b] and the literature cited there.

Franz Reuleaux
(*Eschweiler, 30.09.1829 - 20.08.1905)

Reuleaux was the son of Johann Josef Reuleaux, one of the first steam engine manufacturers in the Rhineland and himself the son of a master engineer from Liège, and of the daughter of H. Graesser, a coal-mine manager at Eschweiler. The Reuleauxs originally came from Hainaut. He studied 1850-1852 at the Karlsruhe Polytechnical Institute under Franz Redtenbacher, 1852 and 1853 at the universities of Berlin and Bonn. He spent the years 1856 to 1864 as professor of machine design at the Swiss Federal Polytechnical Institute and from then until 1896 as professor of mechanical engineering at the Gewerbeinstitut (to become the Berlin University of Technology) at Berlin.

Reuleaux wrote some 90 papers and 12 books not only on all aspects of engineering, but also on technology and civilization/art, on the German language, as well as travel. His chief works were a handbook of machine design : *Der Constructeur* (Brunswick, 1861, ^{4}1889), translated into English (New York, 1893), French, Swedish, and Russian, as well as : *Theoretische Kinematik : I. Grundzüge einer Theorie des Maschinewesens; II. Die praktischen Beziehungen der Kinematik zur Geometrie und Mechanik* (Brunswick, 1877, ^{2}1900), the first volume of which was translated into English (London, 1876, N.Y., 1963), French and Italian.

Reuleaux's theoretical kinematics, a system of analyzing and classifying machinery, has proven so remarkably durable, that he is regarded as the founder of modern kinematics as an independent discipline. He also affected German engineering and industry by other means; he helped in founding the Mannesmann steelworks, worked for the passage of German patents legislation, was a member of the juries of the world exhibitions at Paris (1867), Vienna (1873) and Philadelphia (1876), and he served as the German commissioner at the World Exhibitions of 1876 1879 and 1881. He was the chief spokesman for German engineers and technologists from about 1870 to 1890. His scope of interests was so broad that he is also regarded as the first true philosopher of the technical sciences, emphasizing the basic interconnections between culture and technology. For the details see [120],[79] and the literature cited there.

Elwin Bruno Christoffel
(*Montjoie = Monschau, 10.11.1829 - 15.03.1900)

Christoffel was the son of Franz Karl Christoffel and Maria Helena, née Engels. His grandfather, Charles Joseph Christoffel (Christophe), born 1746 in Verviers, has been traced back in the agnatic line to a Léonard de Faymonville (*c. 1565), in

another branch as far as the eleventh century, to the Carolingian knight Michel d'Awir (near Liège). He was educated at the University of Berlin (1850-1854), where his teachers included Dirichlet, Borchardt, Einsenstein, Steiner, Dove and Sonnenschein. But Christoffel was first and foremost a student of Dirichlet. He received his doctorate in Berlin in 1856 and his Habilitation-degree in 1859. He held professorships at the University of Berlin (1859-1862), the Swiss Federal Polytechnical Institute (1862-1869), the Gewerbeinstitut at Berlin (1869-1872), and at the University of Strasbourg (1872-1894).

His collected works, comprising some 35 articles, appeared under E. B. CHRISTOFFEL : *Gesammelte Mathematische Abhandlungen. Edited by L. Maurer, with assistance of A. Krazer and G. Faber. Teubner, Leipzig 1910, 2. Vols.*

His major contributions lie in diverse areas of mathematics and the physical sciences : (i) Geometry (Christoffel's theorem on determination of a convex surface from its mean radius of curvature), Differential Geometry and Theory of Invariants. His work on the foundations of tensor analysis, associated with the Christoffel symbols (= connection in manifolds), the Riemann-Christoffel curvature tensor, the Christoffel reduction theorem, etc., led to Einstein's General Theory of Relativity and the basic laws of present-day physics; (ii) Function Theory (Christoffel-Schwarz transformation on conformal mappings, abelian integrals, theta functions); (iii) Numerical Analysis (Gauss-Christoffel quadrature formula, Christoffel numbers, functions); (iv) Orthogonal Polynomials and Continued Fractions (Christoffel-Darboux summation formula). This work led to theory of moment problems and Padé approximation; (v) Ordinary and Partial Differential Equations; Potential Theory; (vi) Theory of Shock Waves; and finally, (vii) Dispersion of Light; Mechanical Ether Theory.

His students include R. R. Fujisawa, Ludwig Maurer, Josef Wellstein and Paul Epstein, who became university professors at Tokyo, Tübingen, Strasbourg and Frankfurt, respectively. Fujisawa and some of his renowned students were responsible for raising the standard of mathematics in Japan to the European level. For further details, see the Festschrift [21] edited by Butzer-Fehér, which contains some sixty papers treating various aspects of Christoffel's work.

8. CONCLUDING REMARKS

The question can be raised as to whether the mathematics of the region has specific characteristics.

As we have shown in the first part of this presentation, the school of Liège of the 11th century was justifiably noted for its « pure » mathematics, as represented by elementary geometry and arithmetics, in contrast to the problems of chronology dealt with elsewhere. During the 17th century, Sluse was a notable precursor of differential and integral calculus, but he stands out as an individual rather than as a representative of a regional school. The second period of the Aachen-Liège-Maastricht region that was distinctive in mathematical terms was the 19th century, which can be characterized from four major perspectives :

Firstly, the mathematics of this formative period was primarily « applicable », that is, associated with mathematical analysis, mathematical physics and mechanics. This is definitely the case with Christoffel, Reuleaux, Grandorge and Ronkar. Although Dirichlet was also famous for his number theory, his chief contribution here is his founding of *analytic* number theory. Further, the mathematics of the

second half of his career was mainly inspired by physical problems. Le Paige began his work in geometry, but by 1896 he was director of the Institute of Astro-physics at Liège. Delbœuf was interested in non-Euclidean geometry and in the mechanics of the universe. The field of algebra, as represented elsewhere by figures such as Sylvester, Cayley, Kummer or Kronecker, found no place in the research of the mathematicians of the study region.

Secondly, in contrast to the specialized focus of many mathematicians of the 19th century, Dirichlet, Reuleaux, Christoffel, and Le Paige are noteworthy for their broad range of interests. A later example of this quality can also be cited : Peter J. W. Debeye (*Maastricht, 1884-1966), began his work in mathematical and theoretical physics, then turned to experimental physics and finally chemistry, for which he received the Nobel prize in 1936.

Thirdly, the scholars of the region were regarded as great teachers in their respective fields. The university lectures of Dirichlet, Christoffel and Reuleaux deserve special mention.

Fourthly, most of the scholars of this period not only played a respected role in their universities but also in academic life throughout their countries. This is especially so in the case of Reuleaux, the chief spokesman for German engineers for some twenty years. But it also applies to Dirichlet and Le Paige, as well as to the political economist Wilhelm Lexis (*Eschweiler, 1837-1914), who was one of the founders of mathematical statistics, and the most influential adviser of the Prussian minister of education from 1893 to about 1910.

9. NOTES

The work of Sluse also influenced mathematics in Liège for about a century. Quite a number of books on practical geometry and arithmetic, on an elementary basis however, appeared. Their authors included :

1) Henry Mulkeman (*Liège, c. 1635), professor of geometry, was the author of *Arithmetique theorique et pratique, etc.* (Liège 1671, 333 pp.).

2) Erasme Mulkeman, perhaps the son of Henry, was the author of (i) *Planimetrie ou arpentage nouvellement mise en lumiere* (H. Hoyoux, Liège 1694, 142 pp.); (ii) *Nouvelle pratique d'arithmétique d'une methode tres-facile* (Liège 1698, 538 pp.); (iii) *Abrégé d'arithmétique, etc.* (Liège 1709, 132 pp.). All three books are dedicated to Pierre-Louis de Sluse, brother of R.-F. de Sluse. See [70a, p. 101].

3) Nicolas Cloesket (Cloesquet) (*Liège, c. 1659), professor of languages, arithmetic and geometry, was the author of (i) *Eruditions fructuenses et très-utiles de la vraie et parfaite arithmétique* (Liège, c. 1685); (ii) *Les fleurs et pensées géométriques* (Liège, c. 1700); (iii) *La vraie et parfaite arithmétique des negocians composés* (Liège, c. 1717). The second book, which contains a portrait of R.-F. de Sluse, was dedicated to him; see [70a, pp. 101 f.].

4) Martin Malte (*Liège, c. 1650-1720), professor of arithmetic and geometry, general surveyor and examiner of the surveyors of the city and district of Liège, was the author of (i) *Arithmétique nouvelle, etc.* (Liège 1705, 171 pp.); (ii) *Traité de géométrie servant de reglement, etc.* (Liège, c. 1716); (iii) *Recueil des mesures usitées dans le pays de Liège, etc.* (Urbain Ancion, Liège 1720, 80 pp.) [10, vol. 13, p. 268 f.], [70a, p. 102].

5) Nicolas Martel (*Liège, 1672-1745), secular priest and mathematician, was the author of *Eclaircissement très ample sur les règles fondamentales de l'arithmétique* (Urban Ancion, Liège 1717, 2 Volumes) [10, vol. 13, p. 869], [70a, p. 102].

Further authors are Laurent Gobart S.J. (*Liège, 1658-1730), author of *Tractatus philosophicus de barometro* (Amsterdam 1703, 188 pp.), Joseph Phillipi, school teacher in Liège c. 1715, Jean Harroy, geometer and authorized surveyor c. 1750, as well as Paschal Simonon (†c. 1793); see [70a, p. 103], [10].

These few facts as well as Sections 5 and 6 reveal that much work is still to be done especially in the case of the 16th to 18th century in the entire study region. This was also emphasized by Monsieur Halleux.

Concerning the educational system in Maastricht, around 1500 there were two chapter schools, that attached to the St. Servatius Cathedral and to St. Mary's convent. In 1516 the first Gymnasium was founded, perhaps by the Brethern of the Common life. It was closed in 1532 due to lack of pupils, then reopened from 1551 to 1554. In 1575 the Jesuits reopened it again until forced to leave Maastricht in 1578. Upon their return, the Jesuits reastablished it 1583. The number of pupils rose from 300 in 1583 to 600 in 1595, and 700 in 1619. In the meantime, the two chapter schools had closed. In 1638, when the Jesuits were again forced to leave, the Gymnasium closed, at least until their return in 1673. In the meantime, St. Servatius reastablished its school in 1640. In 1632 the Protestants of the city founded their own Latin school, which ranked as a college from 1683 to 1794; it was devoted to the teaching of theology, philosophy and rhetorics, later also of medicine, geography, and astronomy.

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