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FERMETURES MULTIPLICATIVES

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SUMMARY

A closure (operator) φ of a partially ordered groupoid G is multiplicative if and only if it satisfies the condition :

$$(ab)\varphi = (a\varphi)(b\varphi), \ \forall a, b \in G.$$

When G has a zero, φ is said to be normalized if and only if $0\varphi = 0$.

We designate by F_φ the set of all elements of G invariant under φ. $\Phi(G)$, $\Phi_m(G)$ and $\Phi^0_m(G)$ are respectively the sets of all closures, multiplicative closures and normalized multiplicative closures definable on G.

We characterize the multiplicative closures by means of F_φ. If φ_S denotes the closure defined by the relatively $\wedge$-complete subsemilattice S of the $\wedge$-semilattice L (resp. distributive $\wedge$-semilattice L), $\varphi_S \in \Phi_m(L)$ if and only if $\forall x, y \in L$ and $\forall z \in S$ such that $x \wedge y \leqslant z$ (resp. $x \wedge y = z$), there exists $x_1 \in S$ such that $x_1 \geqslant x$ and $x_1 \wedge y \leqslant z$ (resp. $x_1 \wedge y = z$).

In a modular lattice, $\forall \varphi \in \Phi_m$, F_φ is semiconvex, i. e., it contains x and y whenever it contains $x \wedge y$ and $x \vee y$.

In a negatively ordered semigroup G with zero element, $\forall \varphi \in \Phi^0_m(G)$, the set-complement of a maximal d-ideal is φ-closed (i. e., it contains $x\varphi$ whenever it contains x).

In a lattice L with 0, $\forall \varphi \in \Phi_m(L)$ and $\forall x \in L$, $x\varphi = x \vee 0\varphi$ if L is either section complemented, or relatively semicomplemented, or simple.

If L is a complete lattice, $\Phi(L)$ has the same property. Generally, $\Phi_m(L)$ is not a sublattice of $\Phi(L)$ but this becomes true if L is weakly complemented. In such a lattice, $\Phi_m(L)$ is an Abelian semigroup for the composition of mappings.